

## **DOWNSIZING, CAPITAL INTENSITY, AND LABOR PRODUCTIVITY**

Swapan Sen<sup>\*</sup> and Rezeian Farzin<sup>\*\*</sup>

### **Abstract**

Recent studies on corporate downsizing attribute variability in labor employment to business cycle and the process of technological innovation (Caballero & Hammour, 1996), the outsourcing decision by corporations (Sen & Zhu, 1996), the lack of flexibility of workers compensation schemes (Gerhart & Milkovich, 1990), and institutional factors (Nagao, 1995). This paper presents a simple model to study employment variability due to fluctuations in labor productivity. The study of a production function demonstrates that the firm's use of a factor of production is inversely related to the productivity of that factor. Further, the variability of employment of a factor due to its *own* productivity fluctuations is magnified by the intensity of employment of the *other* factor of production.

### **INTRODUCTION**

Corporate downsizing has led to losses of jobs in millions in large U.S. corporations (Uchitelle & Kleinfield, 1996). Although corporate downsizing is symptomatic of a recession, American firms are laying off more and more workers in spite of a recovering economy when overall unemployment is at historical lows. This has led to the belief that jobs are changing rather than disappearing (Greenberg, 1996). Downsizing and job insecurity have trended up along with rising labor productivity (Filardo, 1995). Such empirical observations would suggest a relationship between downsizing and productivity changes. This relationship, however, is not discernible in the literature. Explanations of employment variability in recent literature appear due to the following sources:

#### **Downsizing As A Result Of Corporate Restructuring**

According to this view, job reallocation results from product and process innovations. As new production processes replace old processes, job creation in new processes and job destruction in old processes becomes necessary outcomes in an evolving economy. Caballero & Hammour (1996) model this Schumpeterian concept of "creative destruction" (Schumpeter 1942) whereby technological progress is associated with massive factor reallocation. They show that during a recession, job creation and job destruction are "coupled" in an efficient economy and "de-coupled" in an inefficient economy. Variability in employment in their model arises as a result of innovative changes in production processes.

In addition, Sen & Zhu (1996) show that firms' outsourcing decisions are complementary to their decisions to downsize. Firms seek reduction of fixed costs through outsourcing as the later is used as a means of reducing headcount in managerial ranks. As a result jobs are reallocated from the original producer to the outside producer, leading to employment variation in both firms.

#### **Downsizing As An Institutional Development**

Nagao (1995) posits downsizing as a result of the conflict between economic rationality and sociopolitical rationality. Economic rationality is associated with maximizing output and long-term profits by the firm. Sociopolitical rationality entails commitment to widely accepted societal values, manifested in general rules of conduct, regarding fair and proper treatment of individuals employed in the business organization. Nagao points out

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\*Christopher Newport University

\*\*Michigan Technological University

that over long term, large business organizations require a balance between economic rationality and sociopolitical rationality and any significant asymmetry causes organizational dysfunction including loss of employment.

### Downsizing Due To Compensation Systems

Gerhart & Trevor (1996) examine organizational differences in the impact of compensation systems on employment variability and suggest that a company's degree of emphasis on variable pay can determine its employment variability in several ways. First, the decision whether or not to reduce labor costs by engaging in layoffs can depend upon the extent of flexibility built into the pay system. Because performance based labor costs are by definition tied to business outcomes, laying off flexibly paid employees during downturns may be less attractive than laying off labor who have fixed wages. Second, the decision to downsize can also depend upon whether a variable pay design encourages decision-makers to emphasize a long-term view of organizational performance. Decision-makers can weigh the short-term labor cost savings of high employment variability versus potential long-term benefits associated with employment stability.

Sampling 152 organizations from the COMPUSTAT database, Gerhart & Trevor find that organizations relying more heavily on long-term compensation incentives for managers exhibited less employment variability. Also, when groups of employees were covered by variable pay plans, their employment variability was observed to be lower.

In summary, the studies cited above provide three different explanations of downsizing; innovative processes, institutional development, and labor compensation schemes. In this paper, we develop a simple model of employment variability and find a basic explanation of employment variation in the fluctuations of factor productivity. This is consistent with the study of innovative processes as firms eliminate less productive processes and workers and retain more productive workers and products. Section II develops a production function model to study the relationship between factor productivity and employment variability and Section III summarizes the results.

## THE MODEL

Define marginal productivity of labor as:

Equation 1

$$\alpha = \frac{\Delta Q}{\Delta L}$$

and average productivity of labor as:

Equation 2

$$\beta = \frac{Q}{L}$$

The production function is given by  $Q=F(K,L)$  where  $Q/K=f(L/K,I)$  is the intensive production function. Along the production function, any one factor-ratio can be expressed in terms of another. Thus:

Equation 3

$$\frac{Q_2}{K_2} = \frac{Q_1}{K_1} + \frac{\Delta Q}{\Delta K}$$

where  $\frac{Q_1}{K_1}$  corresponds to the factor ratio  $\frac{L_1}{K_1}$  and  $\frac{Q_2}{K_2}$  corresponds to the factor ratio  $\frac{L_2}{K_2}$ .

Multiplying both sides of (1) by  $\frac{\Delta L}{\Delta K}$  we obtain:

Equation 4

$$\frac{\Delta Q}{\Delta K} = \alpha \frac{\Delta L}{\Delta K}$$

Similarly, equation (2), when multiplied in both sides by  $\frac{L}{K}$ , yields:

Equation 5

$$\frac{Q_1}{K_1} = \beta_1 \frac{L_1}{K_1}$$

Utilizing (4) and (5) we obtain the following<sup>1</sup>:

Equation 6

$$\frac{Q}{K} = \beta_1 \frac{L_1}{Q_1} + \alpha \frac{\Delta L}{\Delta K}$$

The intensive production function is of the form:

Equation 7

$$\frac{Q}{K} = \left( \frac{L}{K} \right)^n$$

For  $n=1$ , the curve is linear. If  $n>1$ , then  $\left( \frac{Q}{K} \right)^n > 0$  and the curve would be convex. If  $n<1$ , then  $\left( \frac{Q}{K} \right)^n < 0$  and the curve would be concave. Substitution of equation (7) in (6) yields:

Equation 8

$$\beta_1 \frac{L_1}{K_1} + \alpha \frac{\Delta L}{\Delta K} = \left( \frac{L}{K} \right)^n$$

The firm chooses a factor ratio subject to the intensive production function.

Knowing that  $\frac{Q_1}{K_1} = \frac{Q_2}{K_2} - \frac{\Delta Q}{\Delta K}$ , (8) can be rewritten as:

Equation 9

$$\beta_1 \left( \frac{L}{K} - \frac{\Delta L}{\Delta K} \right) + \alpha \frac{\Delta L}{\Delta K} = \left( \frac{L}{K} \right)^n$$

Simplifying the above, we get:

Equation 10

$$\beta_1 \frac{L}{K} + (\alpha - \beta_1) \frac{\Delta L}{\Delta K} = \left( \frac{L}{K} \right)^n$$

Equation (10) is a linear differential equation of the Bernoulli type. To solve (10) we let a dummy term  $v$  to be equal to:

Equation 11

$$v = \beta_1 \left( \frac{L}{K} \right)^{1-n}$$

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<sup>1</sup> For reasons of simplicity we drop subscripts of  $L_2$ ,  $K_2$   $Q_2$ .

Differentiating (11) we obtain:

Equation 12

$$\frac{dv}{dL} = \beta_I (I-n) \left(\frac{L}{K}\right)^{-n} \left(\frac{I}{K}\right)$$

or,

Equation 13

$$dL = \frac{I}{I-n} \frac{I}{\beta_I} \left(\frac{L}{K}\right)^n K dv$$

Dividing both sides of (10) by  $\left(\frac{L}{K}\right)^n$  we get:

Equation 14

$$\beta_I \left(\frac{L}{K}\right)^{I-n} + (\alpha - \beta_I) \frac{\Delta L}{\Delta K} \left(\frac{L}{K}\right)^{-n} = I$$

Substitution of (11) and (13) in equation (14) yields:

Equation 15

$$v + \left(\frac{\alpha - \beta_I}{\beta_I}\right) \left(\frac{I}{I-n}\right) K \frac{dv}{dK} = I$$

Dividing both sides by  $\left(\frac{\alpha - \beta_I}{I-n}\right) \left(\frac{I}{I-n}\right) K$ :

Equation 16

$$\frac{dv}{dK} + \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K} v = \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K}$$

(16) is a linear differential equation of degree one. Its solution takes the following form:

Equation 17

$$v = \left(\exp^{\int P dK}\right)^{-1} \int \left(\exp^{\int P dK}\right) Q dK + c \left(\exp^{\int P dK}\right)^{-1}$$

where  $P = \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K}$  and  $Q = \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K}$ .

We solve (16) by letting a dummy variable y be equal to:

Equation 18

$$y = e^{\int \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K}}$$

Integrating:

Equation 19

$$y = \exp\left(\frac{\beta_I}{\alpha - \beta_I}\right)(I-n)\ln|K|$$

and simplifying (19):

Equation 20

$$y = K^{\frac{\beta_I}{\alpha - \beta_I}}(I-n)$$

Multiplying both sides of (16) by (20), we obtain:

Equation 21

$$K^{-\frac{I-n}{I}} \frac{d}{dK} + K^{-\frac{I-n}{I}} \left( \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K} \right) y = K^{-\frac{I-n}{I}} \left( \frac{\beta_I}{\alpha - \beta_I} (I-n) \frac{I}{K} \right)$$

We summon the terms on the left-hand side as a differential and simplify the right hand side as follows:

Equation 22

$$\frac{d}{dK} \left( K^{\frac{\beta_I}{\alpha - \beta_I}}(I-n) y \right) = K^{\frac{\beta_I}{\alpha - \beta_I}}(I-n) \frac{\beta_I}{\alpha - \beta_I} (I-n)$$

Integrating both sides yields:

Equation 23

$$\frac{\alpha(I-n)}{K^{\alpha - \beta_I}} y = K^{\frac{\beta_I(I-n)}{\alpha - \beta_I}} + c$$

Dividing both sides by  $K^{\frac{\beta_I(I-n)}{\alpha - \beta_I}}$  :

Equation 24

$$y = I + c \left( K^{-\frac{\beta_I}{\alpha - \beta_I}}(I-n) \right)$$

Substituting equation (12) in (24):

Equation 25

$$\beta_I \left( \frac{L}{K} \right)^{I-n} = I + c \left( K^{-\frac{\beta_I}{\alpha - \beta_I}}(I-n) \right)$$

Rewriting (25) and simplifying, we get:

Equation 26

$$\frac{L}{K} = \left( \frac{I + cK \frac{-\beta_l}{\alpha - \beta_l} (I - n)}{\beta_l} \right)^{\frac{I}{I - n}}$$

When marginal and average productivity of labor are equal:

Equation 27

$$\lim_{\alpha \rightarrow \beta_l} \left( \frac{L}{K} \right) = \lim_{\alpha \rightarrow \beta_l} \left( \frac{I + cK \frac{-\beta_l}{\alpha - \beta_l} (I - n)}{\beta_l} \right)^{\frac{I}{I - n}}$$

Which gives:

Equation 28

$$\frac{L}{K} = \left( \frac{I}{\beta_l} \right)^{\frac{I}{I - n}}$$

Thus, the average productivity of labor,  $\beta$ , and the marginal returns,  $n$ , endogenously determine the factor ratio,  $L/K$ . Equation (28) allows deriving the following propositions:

**Proposition 1:** *Each  $L/K$  on the intensive production function has a corresponding average factor productivity that displays an inverse relationship with  $L/K$ .*

*Proof:* When a firm employs new labor whose marginal productivity is equal to the average productivity of all labor the firm already employs, acquiring this new labor will not affect average productivity. Thus, the expression (28) can be rewritten with  $\beta$  showing the averaging productivity of the factor ratio,  $L/K$ :

Equation 29

$$\frac{L}{K} = \left( \frac{I}{\beta} \right)^{\frac{I}{I - n}}$$

Equation (29) demonstrates that average productivity ( $\beta$ ) is inversely related to factor intensity ( $L/K$ ).

Recall from microeconomic theory that marginal productivity equals average productivity at the maximum of average productivity. The factor ratio obtained in the limit, when  $\alpha \rightarrow \alpha_1$ , such as the factor ratio in (28), is an optimum factor ratio.

**Proposition 2:** *Greater is the variability in the employment of a factor, greater the intensity of use of the other factor of production, keeping the other factor constant.*

*Proof:* Suppose there are two firms, A and B. Firm A has high capital intensity and firm B has low capital intensity ( $K_A \gg K_B$ ). Both firms maintain the same absolute amount of capital at any time:

$$(K_A)_0 = (K_A)_1$$

$$(K_B)_0 = (K_B)_1$$

Firms A and B initially have the same labor/capital ratio:

$$\left(\frac{L_A}{K_A}\right)_0 = \left(\frac{L_B}{K_B}\right)_0$$

Average productivity of firm A and B at time 0 are given by  $\beta_A$  and  $\beta_B$

At time 0 the labor/capital ratio of the firms are given by expression (28):

$$\left(\frac{L_A}{K_A}\right)_0 = \left(\frac{I}{\beta_A}\right)_0^{\frac{1}{1-n}}$$

$$\left(\frac{L_B}{K_B}\right)_0 = \left(\frac{I}{\beta_B}\right)_0^{\frac{1}{1-n}}$$

Initially,  $\beta_A = \beta_B$ . The changes in average productivity are represented by  $(\alpha_A - \beta_A)$  and  $(\alpha_B - \beta_B)$ . A change in average productivity is only feasible when  $(\alpha - \beta) < 0$ . Both firms undergo the same changes in average productivity of labor from time 0 to 1:

$$(\alpha_A - \beta_A) = (\alpha_B - \beta_B)$$

The labor/capital ratios of both firms at time 1 are given by expression (26):

$$\left(\frac{L_A}{K_A}\right)_1 = \left(\frac{I + cK_A \frac{\beta_A}{\alpha_A - \beta_A}^{(1-n)}}{\beta_A}\right)^{\frac{1}{1-n}}$$

$$\left(\frac{L_B}{K_B}\right)_1 = \left(\frac{I + cK_B \frac{\beta_B}{\alpha_B - \beta_B}^{(1-n)}}{\beta_B}\right)^{\frac{1}{1-n}}$$

Variability of employment of firm A is shown by:

$$\left(\frac{L_A}{K_A}\right)_1 - \left(\frac{L_A}{K_A}\right)_0 = \left(\frac{I + cK_A \frac{\beta_A}{\alpha_A - \beta_A}^{(1-n)}}{\beta_A}\right)^{\frac{1}{1-n}} - \left(\frac{I}{\beta_A}\right)_0^{\frac{1}{1-n}}$$

Since  $K_A \gg 0$ , the variability of employment of firm A is reduced to:

$$\left(\frac{cK_A \frac{\beta_A}{\alpha_A - \beta_A}^{(1-n)}}{\beta_A}\right)^{\frac{1}{1-n}} - \left(\frac{I}{\beta_A}\right)_0^{\frac{1}{1-n}} \gg 0$$

Similarly, the variability of employment of Firm B:

$$\left(\frac{L_B}{K_B}\right)_0 - \left(\frac{L_B}{K_B}\right)_1 = \left(\frac{I + cK_B \frac{\beta_B}{\alpha_B - \beta_B}^{(1-n)}}{\beta_B}\right)^{\frac{1}{1-n}} - \left(\frac{I}{\beta_B}\right)_0^{\frac{1}{1-n}}$$

Since  $K_B \equiv 0$ , the variability of employment of firm B reduces to:

$$\left(\frac{I}{\beta_B}\right)^{\frac{1}{1-n}} - \left(\frac{I}{\beta_B}_0\right)^{\frac{1}{1-n}} \equiv 0$$

**Proposition 3:** For given changes in  $\mathbf{b}$ , the shift in employment over time is continuous, double differentiable, and monotonic.

*Proof:* When average productivity changes as a linear function of time such as  $\beta = \beta_0 + bt$ , where  $b$  is the slope of the function, (29) can be written as:

$$\frac{L}{K} = \left(\frac{I}{\beta_0 + bt}\right)^{\frac{1}{1-n}}$$

let  $c = \frac{1}{1-n}$  and  $n < 1$ .

Equation 30

$$\frac{L}{K} = \left(\frac{I}{(\beta_0 + bt)^c}\right)$$

This is a polynomial function of degree  $c > 1$ . First and second order differentials are:

$$\frac{\partial \left(\frac{L}{K}\right)}{\partial t} = -bc(\beta_0 + bt)^{-c-1}$$

$$\frac{\partial^2 \left(\frac{L}{K}\right)}{\partial^2 t} = -b^2 c(c-1)(\beta_0 + bt)^{-c-2}$$

It shows that  $\frac{\partial^2 \frac{L}{K}}{\partial^2 t}$  will not change sign if  $\beta = \beta_0 + bt > 0$  and  $c > 2$ . Thus (30) is a monotone smooth function.



## SUMMARY OF RESULTS AND CONCLUSIONS

This study focused on the variation of factor employment and its relationship with factor productivity and presented three main results:

- 1) Variations of factor employment exhibit inverse relationship with factor productivity.
- 2) Larger the capital employed by a firm, higher is its variation in labor employment due to productivity changes at a given level of capital. We find that similar changes in average productivity produces larger variations in employment for a firm which has higher capital intensity, i.e., a firm that employs relatively more capital than labor. This result is symmetric for capital, holding labor employment constant. That is, a firm having higher labor intensity will have higher variation in capital for a given change in the productivity of capital.
- 3) Employment adjustments due to productivity fluctuations are smooth and monotonic.
- 4) Whereas the first result adds to the recent literature on employment variability, the second result lends itself to interesting empirical investigation.

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