Journal of Financial and Strategic Decisions
Volume 12 Number 2 Fall 1999

INVESTMENT HORIZON, DURATION, AND MARKET RATE VARIATIONS: FUNCTIONALITY OF THE NEW-ROR

Kashi Nath Tiwari*

Abstract

Under increasing-rate conditions, the conventional models suggest holding the bond past duration, while under decreasing-rate conditions, these models recommend selling the bond prior to duration to obtain a higher rate of return than that implied by the initial yield to maturity. This appears to be an erroneous result derived from the improper construction of the conventional ROR-models in relation to duration and investment horizon. Under the efficient market hypothesis, it should not matter whether the bond is sold prior to duration or past duration; under all scenarios, the net results should be identical. Only under the conditions of market inefficiency and information asymmetricity, an investor can, intertemporally, gain through the manipulation of the length of the investment horizon. By taking the ratio of value-received and value-paid, this paper constructs an optimal model under which it would not matter whether the bond is held past duration or sold prior to duration. If market rates, on average, remain constant, then the rate of return will be zero; if rates rise, then the investor will experience a negative rate of return; and if market rates fall, then they will experience a positive rate of return. Under the non-averaging procedure of the ratio, even the magnitude of the ROR remains constant over the investment horizon. However, under the ratio-averaging procedure the absolute value of the ROR increases with time such that under falling-rate conditions, the sooner the asset is sold the better, while under rising-rate conditions, the longer the bond is held the better.

THE CONVENTIONAL ROR-MODELS: INFERENCES

Based upon the conventional procedure to calculate the rate of return, the following conclusions can be drawn under changing market rate conditions. If the market rates are expected to rise, then the asset whose duration is less than the investment horizon should be purchased. The longer the investment horizon relative to duration, the higher the rate of return. In order to have the highest rate of return, the investor will buy the asset whose holding point is farthest from the duration.

If the market rates are expected to fall, then an asset whose duration is greater than the investment horizon should be purchased. The shorter the investment horizon relative to duration, the higher the rate of return. In order to obtain the highest rate of return, the investor will buy an asset whose duration is farthest from the asset holding-point.

THE NEW-ROR-MODEL (WITHOUT GEOMETRIC-AVERAGING)

If market rates increase immediately after the asset acquisition and remain at that level throughout the life of the bond, then the new-ROR (R_m) will be a constant negative number for any length of the holding period. This is due to the fact that for a given increase in Y, the ratio of value-received and value-paid will be less than one and the fact that there will be an equi- proportionate change in the value-received and the value-paid at any given point in time It does not matter whether investors hold the asset only for one period or they hold the asset for a longer period of time, they will always experience the same level of negative returns. This is in contrast to the old-model's results where the length of the holding period in relation to duration will determine the value of the old-ROR (R_c) or the positivity and negativity of the yield-adjusted old-ROR. Under conventional models, if $H>D$, then $R_c>Y$ and the yield-adjusted ROR (R_{c-y}) will be positive, while if $H<D$, then $R_c<Y$ and the yield-adjusted ROR (R_{c-y}) will be negative.

*Kennesaw State University

If market rates decrease immediately after the asset acquisition and remain at that level throughout the life of the bond, then the new-ROR (R_m) will be a constant positive number for any length of the holding period. This is due to the fact that for a given decrease in Y, the ratio of value-received and value-paid will be greater than one and the fact that there will be an equi- proportionate change in the value-received and the value-paid at any given point in time It does not matter whether investors hold the asset only for one period or they hold the asset for a longer period of time, they will always experience the same level of positive returns. Under conventional models, if $H>D$, then $R_c<Y$ and the yield-adjusted ROR (R_{c-y}) will be negative, while if $H<D$, then $R_c>Y$ and the yield-adjusted ROR (R_{c-y}) will be positive.

Regardless of the length of the investment horizon (H) relative to duration (D), when market rates remain stationary or change in a neutralizing-fashion, then the New-ROR will be zero, while the old-ROR will be positive and equal to the yield-to-maturity. However, the yield-adjusted Old-ROR will also be zero which will be comparable to the new-ROR results.

THE NEW-ROR-MODEL (WITH GEOMETRIC-AVERAGING)

If the market rates are expected to rise, then the investor should buy the asset whose duration is less than the investment horizon. Under rising market rate conditions, the rate of adjusted-returns will be negative, and this negative rate will be diminishing in value as the asset holding time increases. Therefore, the longer the asset is held, the better.

If the market rates are expected to fall, then the investor should buy the asset whose duration is greater than the investment horizon. Under declining market rate conditions, the rate of adjusted-returns will be positive, and this positive rate will be increasing in value as the asset holding time decreases. Therefore, the shorter the asset is held, the better.

Under geometric-averaging procedure, there appears to be less contradiction between this paper's model and conventional models if the effect of inflation is accounted for: this paper's results do not contradict the intuitive reasoning (or the conventional results to that end). The conventional model is unable to account for the effect of changes in the inflation rate and the opportunity costs, while the proposed model does take into account such effects. Under Conventional models, $R=Y$ at D and $R>Y$ for $H>D$; but is $R>Y$ in reality? The answer is no, if the effect of inflation is taken into account. At D, in the presence of inflation, the inflation-adjusted $R<Y$. As a matter of fact, under rising rate conditions (rising inflation), the inflation-adjusted R will always be less than Y regardless of the holding period [irrespective of whether $H=D$, $H>D$, or $H<D$]. Under inflationary conditions, the inflation-adjusted R will approach Y (Y-R will diminish in value) as the holding period increases. Therefore, under inflationary conditions, the investor should hold the bond as long as possible to minimize the losses in real returns. This is exactly the conclusion of this paper's model as well: [Under rising market rate conditions, the rate of adjusted-returns will be negative, and this negative rate will be diminishing in value as the asset holding time increases. Therefore, the longer the asset is held, the better.].

DISTINGUISHING FEATURES OF THE NEW-ROR MODEL

Under conventional models, the investor's desired result (break-even rate of return) is obtained when $R_c = Y_0$. Under the conventional rate of return models, an investor can obtain the break-even rate of return (i. e., $R_c = Y_0$) under two conditions: (1) if the bond is held till its duration, regardless of whether the market interest rate rises, falls, or remains constant, or (2) if the investment horizon is different from the duration, then the market rates change in an offsetting manner (i.e., market rates on average remain constant).

Under the proposed model, the investor's desired result (break-even rate of return) is obtained when $R_m = 0$. Under the proposed model, an investor will obtain his/her break-even rate of return (i.e., $R_m = 0$) under the following conditions: (1) if rates on average remain constant, regardless of whether the bond is sold prior to duration, at duration, or past duration, (2) if market rates rise, then the bond is held past its maturity with an extendible-maturity clause, or (3) if market rates fall, then the bond is held for the shortest possible time-period with a contractible-maturity clause.

Of course, under both models, a return-maximizing investor will hold the bond as long as possible under rising market rate conditions, while he/she will hold the bond for the shortest possible time period under declining market rate conditions.

Under the proposed model, the investor's break-even rate of return (i.e., $R_m = 0$) is obtained at maturity under constant market rate conditions. Under rising market rate conditions, the rate of return will be always negative during the life the asset; however, the negative rate of return is the lowest at maturity than at any other point in time prior to maturity. Under declining market rate conditions, the rate of return will be always positive during the life of the asset; however, this positive rate of return will be the highest for the lowest holding period.

Under rising market rate conditions, the rate of return is negative and is diminishing in value as the holding period increases. Both at duration and at maturity, the rate of return will be negative. Therefore, to obtain the break-even rate of return, the bond must be held past duration and as long as possible. The break-even rate of return can be obtained only past the maturity, but that is not possible, because an asset cannot be held past its maturity. Therefore, an investor should ask for an extension of the maturity in order to get closer to his/her break-even rate of return. If at the time of the asset-acquisition, market rates are expected to rise, then an extendible-maturity option should be asked for.

Under declining market rate conditions, the rate of return is positive and is increasing in value as the holding period decreases. Both at duration and at maturity, the rate of return will be positive. Therefore, to obtain the break-even rate of return, the bond must be sold before the duration and as quickly as possible. The break-even rate of return can be obtained only prior to time *0*, but that is not possible, because an asset cannot be bought prior to time *0*. Therefore, an investor should ask for a contractible-maturity bond (as short a maturity as possible, and much shorter than the duration of its original issue) in order to get closer to his/her break-even rate of return. If at the time of the asset-acquisition, market rates are expected to fall, then a contractible-maturity option should be asked for.

NOTATION AND DEFINITION

In the ensuing analysis, the following notations will be used:

N = maturity of the asset
H = investment horizon or holding period
Y_i = one-period expected market rate applicable for period i+1, where i = 0, 1, 2, 3, …, n
${}^aYi$ = one-period actual/realized market rate applicable for period i+1, where i = 0, 1, 2, 3, …, n
$P_0 = PV$ = asset price at time 0, (present value of future income stream)
P_n = at time n, the time-value of the original price-paid, P_0
P_h = at time h, the time-value of the original price-paid, P_0
V_n = at time n, the value of the asset earnings
V_h = at time h, the value of the asset earnings
${}^eP_n$ = at time n, the *expected (e)* time-value of the original price-paid, P_0
${}^aP_n$ = at time n, the *actual (a) or realized* time-value of the original price-paid, P_0
${}^eV_n$ = expected (*e*) time-value of asset earnings in n-periods
${}^aV_n$ = *Actual (a) or Realized* time-value of asset earnings at time n
${}^eR_c$ = expected (*e*) rate of return under *conventional (c)* models
${}^aR_c$ = realized rate of return under *conventional (c)* models
${}^eR_m$ = expected (*e*) rate of return under *modified (m)* model of this paper
${}^aR_m$ = realized rate of return under *modified (m)* model of this paper
${}^eR_{ci}$ = at time *i*, expected (*e*) rate of return under *conventional (c)* models
${}^aR_{ci}$ = at time *i*, realized rate of return under *conventional (c)* models
${}^eR_{c-y,i} = {}^eR_c - {}^eR_c = 0$ = at time *i*, yield-adjusted expected ROR under conventional models
${}^aR_{c-y,i} = {}^aR_c - {}^eR_c$ = at time *i*, yield-adjusted actual ROR under conventional models
${}^eR_{mi}$ = at time *i*, expected (*e*) rate of return under the *modified (m)* model of this paper
${}^aR_{mi}$ = at time *i*, realized rate of return under the *modified (m)* model of this paper
$R_{gm,i}$ = at time *i*, the rate of return under the modified model by taking i^{th} root (geometric mean)

Frederick Macaulay's formula for duration is given by:

$$D = \frac{\frac{1C_1}{(1+Y)^1} + \frac{2C_2}{(1+Y)^2} + \frac{3C_3}{(1+Y)^3} + \cdots + \frac{n(C_n + F)}{(1+Y)^n}}{\frac{C_1}{(1+Y)^1} + \frac{C_2}{(1+Y)^2} + \frac{C_3}{(1+Y)^3} + \cdots + \frac{(C_n + F)}{(1+Y)^n}}$$

where D = Duration, N = Maturity, F = Face Value, C_i = Coupon at time i, Y = yield to maturity

Let $C = 100$, $Y_0 = 0.1965$, $F = 1,000$, $N = 5$

Denominator of Duration = $P_0 = 1100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965)$
$+ 100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965 \times 1.1965)$
$+ 100 / (1.1965 \times 1.1965) + 100 / 1.1965 = 709.169095035$

Numerator of Duration = $5 \times 1100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965)$
$+ 4 \times 100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965)$
$+ 3 \times 100 / (1.1965 \times 1.1965 \times 1.1965)$
$+ 2 \times 100 / (1.1965 \times 1.1965) + 100 / 1.1965 = 2836.43165451$

Duration = $2836.43165451 / 709.169095035 = 3.99965491216 \approx 4$

$D \approx 4$

The relationship among ROR, Y, H, and D are presented below for three cases:

Case 1. When market rates remain constant
Case 2. When market rates increase
Case 3. When market rates decrease

Case 1: When market rates remain constant:

Proposition 5 (with or without geometric-averaging**)**: If Y remains constant throughout the life of the bond, then the rate of return under this paper's framework will be 0, while under conventional framework it will be equal to the yield to maturity, but the yield-adjusted-old-ROR will also be zero. This statement holds true regardless of the length of the holding period in relation to duration (i.e., irrespective of whether $H=D$, $H>D$, or $H<D$).

Proof: Suppose that Y remains constant at 19.65%

If $H=D=4$:

$^eV_4 = 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965$
$+ 100 \times 1.1965 + 100 + 1100 / 1.1965 = 1453.4517$

$^aV_4 = 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965$
$+ 100 \times 1.1965 + 100 + 1100 / 1.1965 = 1453.4517$

$^eP_4 = 709.1691$

$^eP_4 = 709.1691 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1453.4517$

$^aP_4 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1453.4517$

$^eR_{c4} = {}^aR_{c4}$

$^aR_{c4} = (^aV_4 / P_0)$^$(1/4) - 1 = (1453.4517 / 709.1691)$^$(1/4) - 1 = 1.1965 - 1 = .1965$ (old-ROR)

$^aR_{m4} = (^aV_4 / P_4) - 1 = (1453.4517 / 1453.4517) - 1 = 0$ (ROR without geometric-averaging)

$R_{gm,4} = (1453.4517 / 1453.4517)$^$(1/4) - 1 = 0$ (ROR with geometric-averaging)

$R_{c-y} = {}^aR_c - {}^eR_c = 0.1965 - 0.1965 = 0$ (yield-adjusted R_c)

Thus if the market rate remains constant, then the ROR will be zero under both the new-model and the old-model; that is, $^aR_{gm,3} = 0$ and $R_{c-y} = 0$.

If (*H=5*) > (*D=4*):

$^{e}V_5 = {}^{a}V_5$

$^{a}V_5 = 100 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 1000 = 1739.05499415$

$^{e}P_5 = {}^{a}P_5$

$^{a}P_5 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1739.05499415$

$^{e}R_{c5} = {}^{a}R_{c5}$

$^{a}R_{c5} = (^{e}V_5 / P_0)^\wedge(1/5) - 1 = (1739.055 / 709.1691)^\wedge(1/5) - 1 = 1.1965 - 1 = .1965$

$^{a}R_{m5} = (^{a}V_5 / P_5) - 1 = (1739.05499 / 1739.05499) - 1 = 0$ (ROR without geometric-averaging)

$R_{gm,5} = (1739.05499 / 1739.05499)^\wedge(1/5) - 1 = 0$ (ROR with geometric-averaging)

$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1965 - 0.1965 = 0$ (yield-adjusted R_c)

Thus if the market rate remains constant, then the ROR will be zero under both the new-model and the old-model; that is, $^{a}R_{gm,3} = 0$ and $R_{c\text{-}y} = 0$.

If (*H=3*) < (*D=4*):

$^{e}V_3 = {}^{a}V_3$

$^{a}V_3 = 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 100 / 1.1965 + 1100/(1.1965 \times 1.1965) = 1214.753$

$^{e}P_3 = {}^{a}P_3$

$^{a}P_3 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 = 1214.75280346$

$^{e}R_{c3} = {}^{a}R_{c3}$

$^{a}R_{c3} = (^{a}V_3 / P_0)^\wedge(1/3) - 1 = (1214.7528 / 709.1691)^\wedge(1/3) - 1 = 1.1965 - 1 = .1965$

$^{a}R_{m3} = (^{a}V_3 / P_3) - 1 = (1214.7528/1214.7528) - 1 = 0$ (ROR without geometric-averaging)

$^{a}R_{gm,3} = (1214.7528 / 1214.7528)^\wedge(1/3) - 1 = 0$ (ROR with geometric-averaging)

$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1965 - 0.1965 = 0$ (yield-adjusted R_c)

Thus if the market rate remains constant, then the ROR will be zero under both the new-model and the old-model; that is, $^{a}R_{gm,3} = 0$ and $R_{c\text{-}y} = 0$.

Case 2: When market rates increase:

Proposition 6a (ROR without geometric-averaging**):** Within the purview of this paper's model, if market rate increases during first year and remains at that level until maturity, then the investor's rate of return will be a constant negative number for any holding period (irrespective of whether $H>D$, $H<D$, $H=D$, $H=0$, or $H=n$). This is in stark contrast to the conventional results, where if market increases then $R>Y$ for $H>D$ (or, old-ROR* > 0), $R<Y$ for $H<D$ (or, old-ROR* < 0), and $R=Y$ for $H=D$ (or, old-ROR* = 0).

Proposition 6b (ROR with geometric-averaging**):** If Y increases, then under the conventional framework, $R=Y$ at duration, $R>Y$ past duration, and $R<Y$ before duration. Under this paper's framework, $R<0$ if Y increases, and the negative rate of return diminishes in value as the holding period increases and approaches maturity. Thus both models appear to produce similar results: if Y increases, then the longer the asset is held the better.

Proof: Suppose that Y increases to 30% from 19.65%

If *H*=*D*=4:

$^{e}V_4 = 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 1100 / 1.1965 = 1453.4517$

$^{a}V_4 = 100 \times 1.30 \times 1.30 \times 1.30 + 100 \times 1.30 \times 1.30 + 100 \times 1.30 + 100 + 1100 / 1.30 = 1464.8538$

$^{e}P_4 = 709.1691$

$^{e}P_4 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1453.4517$

$^{a}P_4 = 709.169095035 \times 1.30 \times 1.30 \times 1.30 \times 1.30 = 2025.45785233$

$^{e}R_{c4} = (^{e}V_4 / P_0)$^$(1/4) - 1 = (1453.4517 / 709.1691)$^$(1/4) - 1 = 1.1965 - 1 = .1965$ (old-ROR)

$^{a}R_{c4} = (^{a}V_4 / P_4)$^$(1/4) - 1 = (1464.8538 / 709.1691)$^$(1/4) - 1 \approx .1965$

$^{a}R_{m4} = (^{a}V_4 / P_4) - 1 = (1464.8538 / 2025.4578) - 1 = - 0.27678$ (ROR without geometric-averaging)

$R_{gm,4} = (1464.8538 / 2025.4578)$^$(1/4) - 1 = - 0.07782$ (ROR with geometric-averaging)

$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1965 - 0.1965 = 0$ (yield-adjusted R_c)

The investment value has depreciated by a total of 27.678% in H-periods or it has depreciated at an annual rate of 7.782%. Thus if the market rate increases, then the new-ROR is negative ($R_{gm,4} < 0$) and the old-ROR* is zero ($R_{c\text{-}y} = 0$).

If (*H=5*) > (*D=4*):

$^{e}V_5 = 100 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 1000 = 1739.05499415$

$^{a}V_5 = 100 \times 1.30 \times 1.30 \times 1.30 \times 1.30 + 100 \times 1.30 \times 1.30 \times 1.30 + 100 \times 1.30 \times 1.30 + 100 \times 1.30 + 100 + 1000 = 1904.31$

$^{e}P_5 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1739.05499415$

$^{a}P_5 = 709.169095035 \times 1.30 \times 1.30 \times 1.30 \times 1.30 \times 1.30 = 2633.09520803$

$^{e}R_{c5} = (^{e}V_5 / P_0)$^$(1/5) - 1 = (1739.055 / 709.1691)$^$(1/5) - 1 = 1.1965 - 1 = .1965$

$^{a}R_{c5} = (^{a}V_5 / P_0)$^$(1/5) - 1 = (1904.31/709.1691)$^$(1/5) - 1 = .2184215435$

$^{a}R_{m5} = (^{a}V_5 / P_5) - 1 = (1904.31 / 2633.095) - 1 = - 0.27678$ (ROR without geometric-averaging)

$R_{gm,5} = (1904.31 / 2633.095)$^$(1/5) - 1 = - 0.6275$ (ROR with geometric-averaging)

$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.2184 - 0.1965 = 0.0219 > 0$ (yield-adjusted R_c)

The investment value has depreciated by a total of 27.678% in H-periods or it has depreciated at an annual rate of 6.275%. Thus if the market rate increases, then the new-ROR is negative ($R_{gm,i} < 0$) and the old-ROR* is positive ($R_{c\text{-}y} > 0$).

If (*H=3*) < (*D=4*):

$^{e}V_3 = 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 100 / 1.1965 + 1100 / (1.1965 \times 1.1965) =1214.753$

$^{a}V_3 = 100 \times 1.30 \times 1.30 + 100 \times 1.30 + 100 + 100 / 1.30 + 1100 / (1.30 \times 1.30) = 1126.81065089$

$^{e}P_3 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 = 1214.75280346$

$^{a}P_3 = 709.169095035 \times 1.30 \times 1.30 \times 1.30 = 1558.04450179$

$^{e}R_{c3} = (^{e}V_3 / P_0)$^$(1/3) - 1 = (1214.7528/709.1691)$^$(1/3) - 1 = 1.1965 - 1 = .1965$

$^{a}R_{c3} = (^{a}V_3 / P_0)$^$(1/3) - 1 = (1126.81065 / 709.1691)$^$(1/3) - 1 = .1669$

$^{a}R_{m3} = (^{a}V_3 / P_3) - 1 = (1126.811 / 1558.045) - 1 = - 0.27678$ (ROR without geometric-averaging)

$^{a}R_{gm,3} = (1126.811 / 1558.045)$^$(1/3) - 1 = = - 0.10238$ (ROR with geometric-averaging)

$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1669 - 0.1965 = - 0.0296 < 0$ (yield-adjusted R_c)

The investment value has depreciated by a total of 27.678% in H-periods or it has depreciated at an annual rate of 10.238%. Thus if the market rate increases, then the new-ROR is negative ($R_{gm,i} < 0$) and the old-ROR* is also negative ($R_{c\text{-}y} < 0$).

Case 3: When market rates decrease:

Proposition 7a (ROR without geometric-averaging)**:** Within the purview of this paper's model, if market rate decreases during first year and remains at that level until maturity, then the investor's rate of return will be a constant positive number for any holding period (irrespective of whether $H>D$, $H<D$, $H=D$, $H=0$, or $H=n$). This is in stark contrast to the conventional results, where if market decreases then $R<Y$ for $H>D$ (or, old-ROR* < 0); $R>Y$ for $H<D$ (or, old-ROR* > 0); and $R=Y$ for $H=D$ (or, old-ROR* = 0).

Proposition 7b (ROR with geometric averaging**):** If Y decreases, then under the conventional framework, $R=Y$ at duration, $R<Y$ past the duration, and $R>Y$ before the duration. Under the proposed framework, $R>0$ if Y increases, and the positive rate of return diminishes in value as the holding period increases and approaches maturity. Thus both models appear to produce similar results: if Y decreases, then the shorter the holding period, the better (i.e., the asset should be discarded as quickly as possible).

Proof: Suppose that Y decreases to 10%

If $H=D=4$:

$${}^{e}V_4 = 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 1100/1.1965 = 1453.4517$$

$${}^{a}V_4 = 100 \times 1.10 \times 1.10 \times 1.10 + 100 \times 1.10 \times 1.10 + 100 \times 1.10 + 100 + 1100/1.10 = 1464.1$$

$${}^{e}P_4 = 709.1691$$

$${}^{e}P_4 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1453.4517$$

$${}^{a}P_4 = 709.169095035 \times 1.10 \times 1.10 \times 1.10 \times 1.10 = 1038.29447204$$

$${}^{e}R_{c4} = ({}^{e}V_4 / P_0)^{\wedge}(1/4) - 1 = (1453.4517 / 709.1691)^{\wedge}(1/4) - 1 = 1.1965 - 1 = .1965 \quad \text{(old-ROR)}$$

$${}^{a}R_{c4} = ({}^{a}V_4 / P_4)^{\wedge}(1/4) - 1 = (1464.1 / 709.1691)^{\wedge}(1/4) - 1 \approx .1965$$

$${}^{a}R_{m4} = ({}^{a}V_4 / P_4) - 1 = (1464.1 / 1038.295) - 1 = 0.410101 \quad \text{(ROR without geometric-averaging)}$$

$$R_{gm,4} = (1464.1 / 1038.295)^{\wedge}(1/4) - 1 = 0.089714 \quad \text{(ROR with geometric-averaging)}$$

$$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1965 - 0.1965 = 0 \quad \text{(yield-adjusted } R_c\text{)}$$

The investment value has appreciated by a total of 41.01% in H-periods or it has appreciated at an annual rate of 8.97%. Thus if the market rate decreases, then the new-ROR is positive ($R_{gm,i} > 0$) and the old-ROR* is zero ($R_{c\text{-}y} = 0$).

If ($H=5$) > ($D=4$):

$${}^{e}V_5 = 100 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 \times 1.1965 + 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 1000 = 1739.05499415$$

$${}^{a}V_5 = 100 \times 1.10 \times 1.10 \times 1.10 \times 1.10 + 100 \times 1.10 \times 1.10 \times 1.10 + 100 \times 1.10 \times 1.10 + 100 \times 1.10 + 100 + 1000 = 1610.51$$

$${}^{e}P_5 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965 = 1739.05499415$$

$${}^{a}P_5 = 709.169095035 \times 1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10 = 1142.12391924$$

$${}^{e}R_{c5} = ({}^{e}V_5 / P_0)^{\wedge}(1/5) - 1 = (1739.055 / 709.1691)^{\wedge}(1/5) - 1 = 1.1965 - 1 = .1965$$

$${}^{a}R_{c5} = ({}^{a}V_5 / P_0)^{\wedge}(1/5) - 1 = (1610.51 / 709.169095035)^{\wedge}(1/5) - 1 = .178264315321$$

$${}^{a}R_{m5} = ({}^{a}V_5 / P_5) - 1 = (1610.51 / 1142.124) - 1 = 0.410101 \quad \text{(ROR without geometric-averaging)}$$

$$R_{gm,5} = (1610.51 / 1142.124)^{\wedge}(1/5) - 1 = 0.071149 \quad \text{(ROR with geometric-averaging)}$$

$$R_{c\text{-}y} = {}^{a}R_c - {}^{e}R_c = 0.1782 - 0.1965 = -0.0183 < 0 \quad \text{(yield-adjusted } R_c\text{)}$$

The investment value has appreciated by a total of 41.01% in H-periods or it has appreciated at an annual rate of 7.1%. Thus if the market rate decreases, then the new-ROR is positive ($R_{gm,i} > 0$) and the old-ROR* is negative ($R_{c-y} < 0$).

If (*H=3*) < (*D=4*):

$^{e}V_3 = 100 \times 1.1965 \times 1.1965 + 100 \times 1.1965 + 100 + 100 / 1.1965 + 1100/(1.1965 \times 1.1965) = 1214.753$

$^{a}V_3 = 100 \times 1.10 \times 1.10 + 100 \times 1.10 + 100 + 100/1.10 + 1100 / (1.10 \times 1.10) = 1331$

$^{e}P_3 = 709.169095035 \times 1.1965 \times 1.1965 \times 1.1965 = 1214.75280346$

$^{a}P_3 = 709.169095035 \times 1.10 \times 1.10 \times 1.10 = 943.904065492$

$^{e}R_{c3} = (^{e}V_3 / P_0)^\wedge(1/3) - 1 = (1214.7528 / 709.1691)^\wedge(1/3) - 1 = 1.1965 - 1 = .1965$

$^{a}R_{c3} = (^{a}V_3 / P_0)^\wedge(1/3) - 1 = (1331 / 709.169095035)^\wedge(1/3) - 1 = .233510218111$

$^{a}R_{m3} = (^{a}V_3 / P_3) - 1 = (1331 / 943.9041) - 1 = 0.410101$ (ROR without geometric-averaging)

$^{a}R_{m3} = (1331 / 943.9041)^\wedge(1/3) - 1 = 0.121373$ (ROR with geometric-averaging)

$R_{c-y} = {}^{a}R_c - {}^{e}R_c = 0.2335 - 0.1965 = 0.0370 > 0$ (yield-adjusted R_c)

The investment value has appreciated by a total of 41.01% in H-periods or it has appreciated at an annual rate of 12.13%. Thus if the market rate decreases, then the new-ROR is positive ($R_{gm,i} > 0$) and the old-ROR* is positive ($R_{c-y} > 0$).

PROPOSED MODEL'S RESULTS: INTUITIVE EXPLANATION

The difference in the results of the two models occur due to the difference in the values of the denominators of the value/price ratio:

$$\text{new-model: } (1+R_m)^h = \frac{value\ received}{value\ paid} = \frac{coupon\ reinvestment\ value + sale\ price}{time value\ of\ price\ paid\ evaluated\ at\ time\ h}$$

$$\text{old-model: } (1+R_c)^h = \frac{value\ received}{value\ paid} = \frac{coupon\ reinvestment\ value + sale\ price}{price\ paid\ at\ time\ 0}$$

New-model when Y↑: An increase in Y increases the value of the denominator of the revenue/price ratio by an amount greater than the change in the value of the numerator such that the ratio becomes less than one, thereby making the ROR negative for any length of the holding period. It does not matter whether the asset is sold prior to duration, at duration or past duration; under all conditions, the ROR will always be negative . Thus regardless of whether the numerator increases (meaning positive coupon reinvestment effect is greater than the negative sale-price effect), decreases (meaning positive coupon reinvestment effect is less than the negative sale-price effect), or remains constant (meaning positive coupon reinvestment effect is equal to the negative sale-price effect), the denominator-effect will always outweigh the numerator-effect, thereby making the ROR negative for any length of the holding period [H∈(0, N)].

New-model when Y↓: A decrease in Y decreases the value of the denominator of the ROR-expression by an amount greater than the change in the value of the numerator such that the revenue/price ratio becomes greater than one, thereby making the ROR positive for any length of the holding period. It does not matter whether the asset is sold prior to duration, at duration or past duration, under all conditions, the ROR will always be positive . Thus regardless of whether the numerator increases (meaning negative coupon reinvestment effect is less than the positive sale-price effect), decreases (meaning negative coupon reinvestment effect is greater than the positive sale-price effect), or remains constant (meaning negative coupon reinvestment effect is equal to the positive sale-price effect), the denominator-effect will always outweigh the numerator-effect, thereby making the ROR positive for any length of the holding period [H∈(0, N)].

Conventional-model when Y↑ An increase in Y has no effect on the denominator of the conventional-model, and therefore, the ROR solely depends upon the new value of the numerator relative to its previous level that existed prior to an increase in Y. The new value of the numerator will be greater than its previous level if the positive coupon-reinvestment effect is greater than the negative sale-price effect (this will occur for all $H>D$, where $R>Y$); the new value of the numerator will be less than its previous level if the positive coupon-reinvestment effect is less than the negative sale-price effect (this will occur for all $H<D$, where $R<Y$); and the new value of the numerator will be equal to its previous level if the positive coupon-reinvestment effect is equal to the negative sale-price effect (this will occur at $H=D$, where $R=Y$).

Conventional-model when Y↓: A decrease in Y has no effect on the denominator of the conventional-model and the ROR solely depends upon the new value of the numerator relative to its previous level that existed prior to a decrease in y. The new value of the numerator will be greater than its previous level if the negative coupon-reinvestment effect is less than the positive sale-price effect (this will occur for all $H<D$, where $R>Y$); the new value of the numerator will be less than its previous level if the negative coupon-reinvestment effect is greater than the positive sale-price effect (this will occur for all $H>D$, where $R<Y$); and the new value of the numerator will be equal to its previous level if the negative coupon-reinvestment effect is equal to the positive sale-price effect (this will occur at $H=D$, where $R=Y$).

RETURNS AT THE INITIAL TIME PERIOD

1. If Y Increases: If y increases during the life of the bond, then regardless of the length of the holding period, the rate of return will be always negative. And if y increases in the very beginning and remains at that level throughout the life of the bond, then the rate of return will be always a constant negative number (e.g., $R = -0.27678$ in the following example) irrespective of whether the holding period is 0, 1, 2, 3, 4, or 5. However, if y increases but by different proportions each year, then the ROR will still be negative, but its value will not be a constant number, it will be different for each year.

Suppose the investor bought the 5-year bond in the morning (when $y=0.1965$, $C=100$, $F=1{,}000$) and wants to sell the bond on the same day in the evening (when $y = 0.30$).

Sale Price at 30% :

$$1100 / (1.3)^5 + 100 / (1.3)^4 + 100 / (1.3)^3 + 100 / (1.3)^2 + 100/1.3 = 512.886$$

Purchase Price at 19.65%:

$$1100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965) + 100 / 1.1965 = 709.1691$$

ROR when market rate increases to 30%:

$$R_{30\%} = 512.886/709.1691 - 1 = -0.27878 < 0$$

However, if the expectations are not realized, then R will not be a constant number. For example, suppose the bond is sold for $512.886 in the evening by using y=30% under the assumption that this 30% rate will remain constant during the life of the asset; however, the next morning suppose the rate changed to 22% and remained that way until maturity. At 22%, the market value of the bond will be:

Sale Price at 22%:

$$= 1100 / (1.22)^5 + 100 / (1.22)^4 + 100 / (1.22)^3 + 100 / (1.22)^2 + 100 / 1.3 = 651.3191$$

ROR when market rate is 22%:

$$R_{22\%} = 651.3191/709.17 - 1 = -0.08158 < 0$$

Had the investor correctly anticipated the rate change to be 22% rather than 30%, his loss would have been only 8.158% instead of 27.678%.

By the same token, if the rate went down to 10% next day rather than remaining constant at 30%, then the investor would have benefited by waiting until next day. The sale price and R under 10% rate are:

Sale Price at 10% :

$$1100 / (1.1)^5 + 100 / (1.1)^4 + 100 / (1.1)^3 + 100 / (1.1)^2 + 100 / 1.1 = 1,000$$

ROR when market rate is 10%:

$$R_{10\%} = 1000 / 709.1691 - 1 = 0.410099$$

Had the investor correctly anticipated the rate change to be 10% rather than 30%, he would have gained by 41% instead of losing by 27.678%.

2. If Y Decreases: If y decreases during the life of the bond, then regardless of the length of the holding period, the rate of return will be always positive. And if y decreases in the very beginning and remains at that level throughout the life of the bond, then the rate of return will be always a constant positive number (e.g., $R=.410099$ in the following example) irrespective of whether the holding period is 0, 1, 2, 3, 4, or 5. However, if y decreases but by different proportions each year, then the ROR will still be positive, but its value will not be of constant number, but it will be different for each year.

Suppose the investor bought the 5-year bond in the morning (when $y=0.1965$, $C=100$, $F=1,000$) and wants to sell the bond on the same day in the evening (when $y=0.10$):

Sale Price when market rate is 10%:

$$=1100 / (1.1)^5 + 100 / (1.1)^4 + 100 / (1.1)^3 + 100 / (1.1)^2 + 100 / 1.1 = 1,000$$

Purchase Price at 19.65%:

$$1100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965 \times 1.1965) + 100 / (1.1965 \times 1.1965) + 100 / 1.1965 = 709.1691$$

$$R_{10\%} = 1000 /709.1691 - 1 = 0.410099$$

However, if the interest rate remained at 10% only on the first day, but beginning the second day and until maturity, the rate remained at 5%, then the investor's sale price and R would have been even higher:

Sale Price at 5%:

$$1100 / (1.05)^5 + 100 / (1.05)^4 + 100 / (1.05)^3 + 100 / (1.05)^2 + 100 / 1.05 = 1216.474$$

$$R_{5\%} = 1000 / 709.1691 - 1 = 0.715349$$

Investor's rate of return would have been 71.5% rather than only 41%, had he sold the bond after one day.

Conversely, if the market rate rose to 30% the next day, then the investor's sale price and R would have been:

Sale Price at 30% :

$$1100 / (1.3)^5 + 100 / (1.3)^4 + 100 / (1.3)^3 + 100 / (1.3)^2 + 100 / 1.3 = 512.886$$

$$R_{30\%} = 512.886/709.1691 - 1 = - 0.27878 < 0$$

Thus the investor was better off by selling the bond in the evening of the same day, rather than waiting for a later time.

CONVENTIONAL MODEL RESULTS: INTUITIVE EXPLANATION

Case 1: If $Y\uparrow$ and $H>D$, then $R>Y$:

Under rising rate conditions, there are two effects:

Negative Price Effect [-]
Positive Coupon Reinvestment Effect [+]

At the point of duration, D:

when $Y\uparrow$, $CRE(+) = PE(-)$, and hence $R = Y$
the two effects are exactly offsetting such that $R = Y$

However, at time $H>D$,

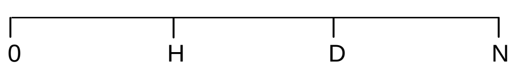
when $Y\uparrow$, $CRE(+) > PE(-)$, and hence $R>Y$
the positive CR-effect dominates the negative price-effect such that $R>Y$

Price Effect (-): Since $H>D$, the bond sold at H will be a shorter-term bond (N-H periods) relative to that being sold at time D (N-D periods), since $(N\text{-}H) < (N\text{-}D)$. For an increase in Y, the negative price-effect on the shorter-term bond will be less pronounced than that on the longer-term bond. Thus by holding the bond past duration, the investor has succeeded in lessening the value of the negative price effect – the negative price effect dwindles as the bond is held for a longer period of time past the duration and will approach zero at maturity.

Coupon Reinvestment Effect (+): Since Y has increased, the longer the bond is held past duration, the greater will be the value of the positive CE-effect. Thus by holding the bond past duration ($H>D$), the investor has succeeded in increasing the value of the positive CE-effect – positive CE-effect accelerates in value as the bond is held for a longer period of time past the duration and will assume the maximum value at maturity.

Thus by holding the bond past duration ($H>D$), the investor will accumulate more of the positive CE-effect and de-cumulate the negative price-effect under rising-rate conditions such that $R>Y$.

Case 2: If $Y\uparrow$ and $H<D$, then $R<Y$:



Under rising rate conditions, there are two effects:

Negative Price Effect [-]
Positive Coupon Reinvestment Effect [+]

At the point of duration, D:

when $Y\uparrow$, $CRE(+) = PE(-)$, and hence $R=Y$
the two effects are exactly offsetting such that $R=Y$

However, at time $H<D$,

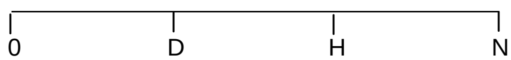
when $Y\uparrow$, $CRE(+) < PE(-)$, and hence $R<Y$
the positive CR-effect is less than the negative price-effect such that $R<Y$

Price Effect (-): Since *H*<*D*, the bond sold at *H* will be a longer-term bond (*N*-*H* periods) relative to that being sold at time *D* (*N*-*D* periods), since (*N*-*H*) > (*N*-*D*). For an increase in Y, the negative price-effect on the shorter-term bond will be less pronounced than that on the longer-term bond. Thus by holding the bond for less than the duration, the investor has increased the value of the negative price effect – the negative price effect increases as the bond is held for a shorter period of time for less than the duration.

Coupon Reinvestment Effect (+): Since *Y* has increased, if the bond is held for a shorter period of time, then the positive *CE*-effect will be less in value. Thus by selling the bond before duration (*H*<*D*), the investor has decreased the value of the positive *CE*-effect – positive *CE*-effect is less pronounced for shorter-period of time.

Thus by selling the bond before duration (*H*<*D*), the investor will have less of the positive *CE*-effect and more of the negative price-effect under rising-rate conditions such that *R*<*Y*.

Case 3: If Y↓ and *H*>*D*, then *R*<*Y*:



Under declining rate conditions, there are two effects:

Positive Price Effect [+]
Negative Coupon Reinvestment Effect [-]

At the point of duration, *D*:

when Y↓, *CRE*(-) = *PE*(+), and hence *R*=*Y*
the two effects are exactly offsetting such that *R*=*Y*

However, at time *H*>*D*,

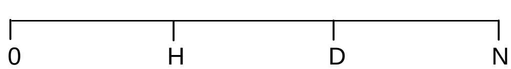
when Y↓, *CRE*(-) > *PE*(+), and hence *R*<*Y*
the negative *CR*-effect is greater than the positive price-effect such that *R*<*Y*

Price Effect (+): Since *H*>*D*, the bond sold at *H* will be a shorter-term bond (*N*-*H* periods) relative to that being sold at time *D* (*N*-*D* periods), since (*N*-*H*) < (*N*-*D*). For a decrease in Y, the positive price-effect on the shorter-term bond will be less pronounced than that on the longer-term bond. Thus by holding the bond past duration, the investor has decreased the value of the positive price effect.

Coupon Reinvestment Effect (-): Since Y has decreased, the longer the bond is held past duration, the greater will be the value of the negative CE-effect. Thus by holding the bond past duration (*H*>*D*), the investor has increased the value of the negative CE-effect.

Thus by holding the bond past duration (*H*>*D*), the investor will accumulate more of the negative CE-effect and less of the positive price-effect under rising-rate conditions such that *R*<*Y*.

Case 4: If Y↓ and *H*<*D*, then *R*>*Y*:



Under declining rate conditions, there are two effects:

Positive Price Effect [+]
Negative Coupon Reinvestment Effect [-]

At the point of duration, *D*:

when Y↓, *CRE*(-) = *PE*(+), and hence *R*=*Y*
the two effects are exactly offsetting such that *R*=*Y*

However, at time $H<D$,

when Y↓, $CRE(-) < PE(+)$, and hence $R>Y$
the negative *CR*-effect is less than the positive price-effect such that $R>Y$

Price Effect (+): Since $H<D$, the bond sold at H will be a longer-term bond ($N-H$ periods) relative to that being sold at time D ($N-D$ periods), since $(N-H) > (N-D)$. For a decrease in Y, the positive price-effect on the shorter-term bond will be less pronounced than that on the longer-term bond. Thus by selling the bond prior to duration, the investor has increased the value of the positive price effect.

Coupon Reinvestment Effect (-): Since Y has decreased, if the bond is sold prior to duration, then the negative CE-effect will be less pronounced. Thus by selling the bond prior to duration ($H<D$), the investor has decreased the value of the negative CE-effect.

Thus by holding the bond prior to duration ($H<D$), the investor will accumulate more of the positive price-effect and less of the negative CE-effect under falling-rate conditions such that $R>Y$.

THE PATH-DEPENDENT ROR

At maturity, the value-received (reinvestment value of asset earnings) can be defined as:

$$^aV_n = C_1(1+{}^aY_1)(1+{}^aY_2)(1+{}^aY_3)+ \ldots +(1+{}^aY_{n-1}) + C_2(1+{}^aY_2)(1+{}^aY_3)(1+{}^aY_4)+ \ldots +(1+{}^aY_{n-1}) \\ + C_3(1+{}^aY_3)(1+{}^aY_4)(1+{}^aY_5)+ \ldots +(1+{}^aY_{n-1}) + \ldots + C_n + F$$

and the value-paid (reinvestment value of the price-paid) can be defined as:

$$^aP_n = P_0[(1+{}^aY_0)(1+{}^aY_1)(1+{}^aY_2)+\ldots+(1+{}^aY_{n-1})]$$

At time H, the value-received (reinvestment value of asset earnings) can be defined as:

$$(CRE)_h + P_h^s$$

where the coupon reinvestment value is given by:

$$(CRE)_h = C_1(1+{}^aY_1)(1+{}^aY_2)(1+{}^aY_3)+ \ldots +(1+{}^aY_{h-1}) + C_2(1+{}^aY_2)(1+{}^aY_3)(1+{}^aY_4)+ \ldots +(1+{}^aY_{h-1}) \\ + C_3(1+{}^aY_3)(1+{}^aY_4)(1+{}^aY_5)+ \ldots +(1+{}^aY_{h-1}) + \ldots + C_h$$

Sale-price at time h is given by,

$$\begin{aligned} P_h^s = {} & C_{h+1}/(1+{}^aY_h) \\ & + C_{h+2}/(1+{}^aY_h)(1+{}^aY_{h+1}) \\ & + C_{h+3}/(1+{}^aY_h)(1+{}^aY_{h+1})(1+{}^aY_{h+2}) \\ & +\ldots+ C_{n-2}/(1+{}^aY_h)(1+{}^aY_{h+1})(1+{}^aY_{h+2}) \ldots (1+{}^aY_h)(1+{}^aY_{h+1})(1+{}^aY_{n-3}) \\ & + C_{n-1}/(1+{}^aY_0)(1+{}^aY_1)(1+{}^aY_2)+ \ldots +(1+{}^aY_{n-2}) \\ & + (C_n+F)/(1+{}^aY_0)(1+{}^aY_1)(1+{}^aY_2)+ \ldots +(1+{}^aY_{n-1}) \end{aligned}$$

Time value of the purchase price is given by,

$$^aP_h = P_0[(1+{}^aY_0)(1+{}^aY_1)(1+{}^aY_2)+\ldots+(1+{}^aY_{h-1})]$$

The rate of return at time h is given by:

$$R_{mh} = [(CRE)_{h\,+}\,{}^aP_h^s\,]\,/\,{}^aP_h - 1 \qquad \text{(without geometric-averaging)}$$

$$R_{gmh} = \{[(CRE)_h + {}^aP_h^s\,]/\,{}^aP_h\}^{(1/h)} - 1 \qquad \text{(with geometric-averaging)}$$

Case 1: For $dY = 0$ and $H \in [0, n]$:

$$[(CRE)_{h} + {}^{a}P_h^{s}] = {}^{a}P_h$$

and

$$d[(CRE)_h + {}^{a}P_h^{s}] / dY_i = d({}^{a}P_h) / dY_i = 0$$

such that

$$R_{mh} = 0$$
$$R_{gmh} = 0$$

for all $h \in [0, n]$

while under conventional models,

$$R_{gmh} = Y \text{ and } R_{c\text{-}y} = 0; \text{ for all } h \in [0, n]$$

Case 2: For $dY > 0$, and $H \in [0, n]$:

$$[(CRE)_h + {}^{a}P_h^{s}] < {}^{a}P_h$$

and

$$d[(CRE)_h + {}^{a}P_h^{s}] / dY_i = d({}^{a}P_h) / dY_i$$

such that

$$R_{mh} < 0$$
$$R_{gmh} < 0$$

however,

$$R_{gmh} < R_{gmh\text{-}i}$$

for all $h \in [0, n]$ and $i \in [1, h]$

$$R_{gmh} > Y \text{ and } R_{c\text{-}y} > 0 \text{ for } H>D$$
$$R_{gmh} < Y \text{ and } R_{c\text{-}y} < 0 \text{ for } H<D$$
$$R_{gmh} = Y \text{ and } R_{c\text{-}y} = 0 \text{ for } H=D$$

Case 3: For $dY < 0$ and $H \in [0, n]$:

$$[(CRE)_h + {}^{a}P_h^{s}] > {}^{a}P_h$$

and

$$d[(CRE)_h + {}^{a}P_h^{s}] / dY_i = d({}^{a}P_h) / dY_i$$

such that

$$R_{mh} > 0$$
$$R_{gmh} > 0$$

however,

$$R_{gmh} > R_{gmh\text{-}i}$$

for all $h \in [0, n]$ and $i \in [1, h]$

while under conventional models,

$$R_{gmh} > Y \text{ and } R_{c\text{-}y} > 0 \text{ for } H<D$$
$$R_{gmh} < Y \text{ and } R_{c\text{-}y} < 0 \text{ for } H>D$$
$$R_{gmh} = Y \text{ and } R_{c\text{-}y} = 0 \text{ for } H=D$$

CONCLUDING REMARKS

Within the purview of this paper's model, the rate of return is not path-dependent when the ratio of value-received and value-paid is not distorted through geometric-averaging. However, if the geometric-averaging of the ratio is allowed, then the rate of return may become path-dependent but only in modest values. Regardless of whether or not the ratio is subjected to geometric-averaging, the results of the new-model introduced in this paper are in stark contrast with those of the conventional ones. A change in the length of the holding period does not change the values of the investment returns. The rate of return is path-dependent in the conventional models in that its value varies in accordance with the variations in the values of market rates, investment horizon, and duration; only under stable market conditions, the rate of return is a known constant number. The new-ROR model introduced in this paper provides a clear and precise evaluation of the effect of changes in investment values.

REFERENCES

1. Bierwag, G.O., G.G. Kaufman, and A.L. Toevs, "Single Factor Duration Models in a Discrete General Equilibrium Framework," *Journal of Finance*, Vol. 37, May 1982, pp. 325-338.

2. Bierwag, G.O., G.G. Kaufman, and A.L. Toevs, "Duration: Its Development and Use in Bond Portfolio Management," *Financial Analyst Journal*, Vol. 39, July-August 1983, pp. 13-35.

3. Bodie, Z., A Kane, and A. Marcus, *Essentials of Investments*, New York: Irwin/McGraw-Hill, Inc., 1998.

4. Chambers, D.R., W.T. Carleton, and R.W. McEnally, "Immunizing Default-Free Bond Portfolios with a Duration Vector," *Journal of Financial and Quantitative Analysis*, Vol. 23, March 1988, pp. 89-104.

5. Cox, J.C., J. Ingersoll, Jr., and S.A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometerica*, Vol. 35, March 1985, pp. 385-407.

6. Daniel, K. and Sheridan Titman, "Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns," *Journal of Finance*, Vol. 52(1), March 1997, pp. 1-34.

7. Fama, Eugene F., and Kenneth R. French, "Size and Book to Market Factors in Earnings and Returns," *Journal of Finance*, Vol. 50, 1995, pp. 131-156.

8. Fama, Eugene F., and Kenneth R. French, "Multifactor Explanation of Asset Market Anomalies," *Journal of Finance*, Vol. 51, 1995, pp. 505-84.

9. Gilchrist, Simon, and Charles Himmelberg, "Evidence on the Role of Cash Flow for Investment," *Journal of Monetary Economics*, Vol. 36, 1996, pp. 541-572.

10. Hearth, D. and J. Zaima, *Contemporary Investments*, Fortworth: Dryden Press, 1998.

11. Ingersoll, J.E., Jr., J. Skelton, and R.L. Weil, "Duration Forty Years Later," *Journal of Financial and Quantitative Analysis*, Vol. 13, November 1978, pp. 627-650.

12. Keown, Arthur. J., *Personal Finance: Turning Money into Wealth*, Englewood Cliffs, NJ: Prentice-Hall, 1998.

13. Leibowitz, Martin L. and A. Weinberger, "The Uses of Contingent Immunization," *Journal of Portfolio Management*, Vol. 8, Fall 1981, pp. 51-55.

14. Macaulay, Frederick R. *Some Theoretical Problems Suggested by the Movements of the Interest Rates, Bond Yields, and Stock Prices in the United States since 1856*, New York: National Bureau of economic Research, 1938.

15. Sias, R.W. and Laura T. Starks, "Institutions and Individuals at the Turn-of-the-Year," *Journal of Finance*, Vol. 52 No. 4, pp. 1543-1562.

16. Weil, Roman L., "Macaulay's Duration: An Appreciation," *Journal of Business*, Vol 46, October 1973, pp. 589-592.