THE STATISTICAL DISTRIBUTION
OF DAILY EXCHANGE RATE PRICE CHANGES:
DEPENDENT VS INDEPENDENT MODELS

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Abstract

This study evaluates recently reported, conflicting, models for the probability distributions of daily exchange rate price changes. The conflicting conclusions arise from differing data sets, noncomparable evaluation criteria, and failures to directly compare the candidate models. This study evaluates the mixed jump diffusion model, a discrete mixture of normals distribution model, and four alternative forms of the generalized autoregressive conditional heteroscedastic (GARCH) model. We estimate parameters for each model using maximum likelihood techniques; the goodness-of-fit for the models is measured using Schwarz's criteria. In contrast to some recently published results, none of our autoregressive conditional variance models dominated the others; also none of these models consistently dominated the two models that assume returns are independent. In the cases where there is significant first-order heteroscedasticity in the data set, the GARCH models are superior only 50% of the time. In the most recent subperiod (Jan 88 - Dec 92) tests show that for three of the four currencies the first-order heteroscedasticities are less pronounced than in prior periods. Curiously, first-order GARCH parameters are significant in cases where tests for first-order heteroscedasticity are not significant; this result suggests that our models may be misspecified. Results indicate that independence should not be overlooked, and future research should not focus on the search for the perfect GARCH model, but attempt to develop models that incorporate the pronounced volatility clustering found in exchange rate price series and the independent behavior that exists in the data. These conclusions are consistent with recent findings related to high frequency (intra-daily) returns.

INTRODUCTION

Researchers continue to seek theoretical probability distribution models that fit the empirical distributions of changes in spot exchange rates. Better theoretical models of these empirical distributions should contribute to more accurate pricing models for exchange rates and improved test statistics for such models.

For years researchers assumed that the empirical distribution of changes in exchange rates was best described by either a normal or lognormal probability distribution. Indeed, some current studies and most current tests "automatically" apply the logarithmic transformation to returns from spot, forward, and futures exchange rate changes; this transformation assumes, explicitly or implicitly, that the transformed data produces returns that are normally distributed. Recent empirical studies reject these assumed probability distributions; these studies do not conclusively agree on any single alternative model. These studies usually support either the mixed jump diffusion model, a discrete mixture of normal distributions(multinormal model), or some type of generalized autoregressive conditional heteroscedastic (GARCH) model.

Boothe and Glassman (1987) evaluate four probability distributions which have been proposed to describe exchange rate changes: the normal, the symmetric Stable Paretian, the Student t, and a mixture of normals. Their results indicate that changes in logs of exchange rates have nonnormal distributions for daily data. The Student distribution and the mixture of two normal distributions provide the best fits.

Tucker and Pond (1988) investigate four generating processes for daily exchange rate changes for six major currencies (1980-1984). The four process are the scaled t-distribution, the general stable distribution, multinormal distributions, and the mixed jump diffusion model. A series of pairwise comparisons favor the mixed jump model

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for every currency tested. In a similar study Akgiray and Booth (1988), found that the mixed jump diffusion model was superior to both discrete mixtures of normal distributions and stable distributions.

Hsieh (1988) found highly significant autocorrelations for the squared residuals of daily data, but an ARCH(12) process accounts for most of the nonlinear stochastic dependencies found. Studies by Taylor (1986), McCurdy and Morgan (1988), and Kugler and Lenz (1990), find that the GARCH(1,1) model has a superior fit than the ARCH(12) model.

Fujihara and Park (1990) test the ARCH model, a multinormal model, the mixed jump diffusion model, and the Student t-distribution model. Their results support the ARCH model for three out of the five currencies examined. Fujihara and Park's results, unlike previous results, reject the mixed jump diffusion model as the best fitting model among the models that assume independence. For a through review of ARCH/GARCH modeling in finance see Bollerslev, Chou and Kroner (1992).

Baillie, Bollerslev, and Mikkelson (1996) introduce the fractionally integrated generalized autoregressive conditional heteroscedastic (FIGARCH) processes. They report that the influence of lagged squared innovations has a slow hyperbolic rate of decay which tends to zero for long lags. They note that this process resembles the fractionally integrated class of processes for conditional means, with similar flexibility in modeling the persistence of shocks to the conditional variance process. Using daily time series of the German Mark/ U.S. dollar exchange rate, they show that the FIGARCH(1,d,1) model is superior to both the GARCH(1,1) and the integrated GARCH (IGARCH(1,1)).

Bollerslev and Ghysels (1996) propose periodic conditional heteroscedastic structure (P-ARCH(1,1), P-GARCH(1,1)) to formulate time series models that capture the repetitive seasonal time variation of the second order moments. They show a loss in efficiency when this periodic factor is ignored.

Although our study is not the first study to examine "in a horse race" whether dependent models describe the data better than independent models. This study is important for several reasons. First, prior research comparing dependent and independent models tested only the dependent ARCH model. In that study the ARCH model is found not to dominate the independent models. The current literature using daily data, post 1990, have focused exclusively on refinements to the dependent GARCH model. Since these models have not been compared to independent models we do not know if the recent movement of research to these purely dependent models is appropriate. This question needs to be addressed since recent research using intra-daily data suggests that although the volatility process may exhibit long-run dependance, significant independence exist in the data (Anderson and Bollerslev (1997))[1].

In this study we evaluate the GARCH(1,1), GARCH-M(1,1), EGARCH(1,1), IGARCH(1,1)[2], mixed jump diffusion and a mixture of 2 normals so as to directly compare alternative dependent vs an independent model.

All variations of the GARCH model are not examined here (For example, P-GARCH and FIGARCH). We focus on the GARCH models that are used the most frequently in the literature when examining financial price series. Also, since variations of the GARCH type models that are not represented in this study have similar forms to the dependent model variations used in this study, the GARCH models examined should adequately represent alternative, similar dependent-form models.

Although proper procedure to fit a GARCH model entails diagnostic checking in addition to estimation we focus on the GARCH(1,1) type processes due to support found for these type of models in previous research.

If shocks persist in the conditional variance process sequential variance will be influenced by such shocks, so as to produce clusters. If return distributions depend on past returns, the GARCH models should dominate the independent models. If variance clustering does not commonly occur, the mixture of normals model or the mixed jump diffusion model will provide a better fit to the data. If we do find that the independent models have a superior fit than the GARCH processes, this would indicate that significant independence exists in the data, and show that the recent movement of the research to purely dependent models is inappropriate.

Another problem with past research is that no common single conventional test criteria is used. Studies have used the Schwarz criteria (SC), Akaike information criteria (AIC), mean square error, correlation integral-based tests, Lung box test, logarithmic loss functions and several others.

To compare models the Schwarz criteria is used in this study. Monte Carlo simulations have been conducted showing the accuracy of this criteria for exchange rate data (For example, see Tucker and Pond (1988) and Akgiray and Booth (1988)). Simulations have not been conducted for the other methods, and to date, no studies have compared agreements or disagreements for these many alternative criteria. Also, when comparing the AIC and SC, the SC has superior large sample properties. As sample size approaches infinity the SC criteria is asymptotically consistent, whereas the AIC is biased towards selecting an overparameterized model.

Prior studies that examine GARCH type process do not include tests for heteroskedasticity. Therefore we do not know if these models are picking up what they are theoretically designed to. We include tests for hereoskedasticity to examine this question.

Below, we describe the general features of the alternative models and the goodness-of-fit criteria. The specific functional forms of the models used in this study follow that discussion. Section III describes the data, estimation techniques, and evaluation criteria. The final section summarizes our results and offers conclusions.

BACKGROUND ON THE PROBABILITY DISTRIBUTION MODELS

If changes in exchange rates are generated by the mixed jump diffusion process, the variance of the primary process doesn't change but the process generates infrequent nonlocal jumps with large variances. Most research on the mixed jump diffusion model assumes that the jumps are independent and have a identical normal distribution. In this case, the mixed jump diffusion model is just a special case of a multinormal model where the Poisson process determines when parameters change.

The discrete mixture of normals model assumes that exchange rate changes are normally distributed at any point in time but the underlying parameters of the normal distributions are assumed to shift over time. With the multinormal model, the mean can change, the variance can change, or both can change. In general although the mean, the variance and the number of distributions are unrestricted, most prior applications of this model assume that the returns are independent. The GARCH model assumes a normal distribution,[3] with a conditional variance that changes over time as a function of past squared deviations from the mean and as a function of past variances. The GARCH models are also multinormal models with some very specific restrictions. However, most applications of the GARCH model assume a constant mean, with variances that can be functions of past squared deviations from the mean and past variances as well. Since the variance of GARCH model is conditional and the mean is constant, it is a highly restrictive multinormal model[4]. Depending on the parameters in the conditional variance equations of the GARCH models, the data can come from up to n normal distributions, where n is the sample size. It should be noted that conditional variance of a GARCH model depends on previous residuals and GARCH models do not belong to a fixed member of the normal distribution family.

CANDIDATE PROCESSES

Discrete Mixtures of Normal Distribution

Models based on discrete mixtures of normal distributions allow for shifts in parameter values amongst N possible sets of parameter values. Kon (1984) provides a through discussion of discrete mixture of normal distributions. McFarland, Pettit, and Sung (1982) and So (1987) strongly support this model as a basis for the observed day-of-the-week effects.

The empirical distribution for the changes in spot exchange rates can be described using a discrete mixture of normals distribution. With this distribution, each outcome for changes in the spot exchange rate at t, (S(t)) has a normal distribution; these normal distributions occur with probability p_i ($I = 1, \ldots K$), where $p_1 + p_2 + \ldots + p_K = 1$.

The probability density function for $S(t)$ is:

Equation 1

$$f(S(t) \mid \Theta) = \sum_{I=1}^{K} p_i N_i (S(t) \mid \mu_i, \sigma_i^2) , \quad -\infty < S(t) < \infty$$

where the parameter vector is $\Theta = (p_1, \ldots, p_K, \mu_1, \ldots, \mu_K, \sigma_1^2, \ldots, \sigma_K^2)$ and the normal density function for the ith distribution is given by $N_i(S(t) \mid \mu_i, \sigma_i^2)$. The mean and the variance of $S(t)$ are given by:

Equation 2

$$E(S(t)) = \sum_{I=1}^{K} p_i \mu_i$$

Equation 3

$$Var(S(t)) = \sum_{I=1}^{K} p_i \ [(\mu_i - E(S(t)))^2 + \sigma_i^2]$$

Mixed Diffusion Jump

Merton (1976) developed a mixed jump diffusion model which explicitly admits jumps in the underlying generating process. Here, we modify Merton's model to apply to exchange rate variations so that the arrival of ordinary information produces exchange rate changes that follow a lognormal diffusion process (geometric Brownian motion). Abnormal information is presumed to be generated by a Poisson process that produces iid normally distributed jumps in exchange rate changes. The sample paths of this process can be described by the following stochastic differential equation:

Equation 4

$$dS(t)/S(t) = \alpha dt + \sigma dB(t) + J_t dP(t)$$

where $S(t)$ = the spot exchange rate at time t, α = the instantaneous conditional expected rate of return per unit time for the Brownian motion part of the process, σ = the instantaneous conditional standard deviation of the Brownian motion part of the process, $B(t)$ = standard Brownian motion, J_t = a normal random variable with mean μ_J and variance σ^2_J representing the logarithm of one plus the percentage change in the price of the Poisson jump at time t, This variable measures the size of the Poisson jump at time t, $P(t)$= is an independent Poisson process with intensity parameter $\tau > 0$, thus, τdt is the probability that a jump will occur in the interval $(t, t + dt)$. The probability density function of $x(t) = log[S(t) / S(t-1)]$ is:

Equation 5

$$f(x(t) \mid \Theta) = \sum_{n=0}^{\infty} (e^{-\tau}\tau^n / n!) N(X \mid \mu + n\mu_J, \sigma^2 + n\sigma^2_J)$$

where $\mu = \alpha - .5\sigma^2$ and $N(\ )$ is the normal density function, with parameter vector $\Theta = (\mu, \sigma^2, \tau, \mu_j, \sigma^2_J)$. Moments of all orders exist. The mean of the distribution is given by:

Equation 6

$$E(X) = \mu + \tau\mu_J$$

and the variance is:

Equation 7

$$Var(X) = \sigma^2 + \tau(\sigma^2_J + \mu^2_J)$$

Generalized Autoregressive Conditional Heteroscedastic (GARCH)

The GARCH model of Bollerslev (1986) allows for the conditional variance to depend upon past information and therefore vary over time. It allows for a more flexible lag structure than the ARCH model of Engle (1982). In the GARCH model the conditional variance is predicted by past forecast errors and past variances. Formally, the model can be expressed as follows:

Equation 8

$$Y(t) = x(t)P + e(t)$$

Equation 9

$$e(t) \mid \Phi_{t-1} \sim N(0, \sigma^2(t))$$

Equation 10

$$\sigma^2(t) = \alpha_0 + \sum_{I=1}^{p} \alpha_i e^2(t - I) + \sum_{j=1}^{q} \beta_i \sigma^2(t - j)$$

where the conditional information set at time *t-1* is denoted Φ_{t-1}. In this study $Y(t)$ is equal to the change in $log(S(t))$, the log of the spot exchange rate. $X(t)$ is a *1×k* vector of lagged endogenous variables included in the information set. *P* is a *k×1* vector of unknown parameters.

Generalized Autoregressive Conditional Heteroscedastic in the Mean (GARCH-M)

In the GARCH-M model the conditional mean is a function of the conditional variance equation. In finance this model may be an improvement over the GARCH model because it is suited to account for the risk-return relationship, where increased returns are expected with increased risk. The GARCH-*M(q,p)* model can be expressed using the GARCH equations (9) and (10) but replacing equation (8) with:

Equation 11

$$Y(t) = x(t)P + K\sigma^2(t) + e(t)$$

where *K*= parameter for the conditional mean part of the process.

Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH)

The EGARCH model of Nelson (1991), has no restrictions on parameters whereas the GARCH model imposes nonnegativity constraints on the parameters α and β. $\sigma^2(t)$ is an asymmetric function of past errors as defined by equations (8), (9), and (12):

Equation 12

$$ln(\sigma^2(t)) = \alpha_0 + \sum_{I=1}^{p} \alpha_i g(z(t-I)) + \sum_{j=1}^{q} \beta_i \, ln \, \sigma^2(t-j)$$

where $g(z(t)) = uz(t) + \beta[|z(t)| - E|z(t)|]$ and $z(t) = e(t)/\sigma(t)$. Equation (12) is similar to an unrestricted ARMA(*p,q*) model for the log of $\sigma^2(t)$. If $\alpha_i u < 0$, the variance will rise (fall) when $e(t-1)$ is negative (positive). If $z(t)$ is assumed to be *i.i.d* normal, $e(t)$ is variance stationary provided all the roots of the autoregressive polynomial (*B*) = *1* lie outside the unit circle.

Integrated Generalized Autoregressive Conditional Heteroscedastic (IGARCH)

The integrated in variance, or IGARCH model, modifies the GARCH model to incorporate an approximate unit root in the variance equation, $\alpha_1 + \ldots + \alpha_p + \beta_1 + \ldots + \beta_q = 1$. The IGARCH resembles the ARIMA class of models for conditional first moments.

DATA, ESTIMATION TECHNIQUES AND EVALUATION CRITERIA

Daily closing spot price data for the U.S. dollar versus British pound, Canadian dollar, German mark, and Japanese yen were obtained from the Chicago Mercantile Exchange Yearbook for 1978 to 1987, and from the Merrill Lynch debt markets group's fixed income research data base for the years 1988 to 1992. The daily series (n=3817) represents changes between business days with no adjustments for holidays. The data set was split into five year intervals to retest all models to evaluate the stability of probability models, over changing time periods; either the form of the model or the parameter values for a selected model form could change over time.

Estimates for each of the candidate models are based on maximum likelihood methods. In this study we use the IMSL subroutine (DNCONG). The Schwarz criteria is used to evaluate the models. This criteria uses Bayesian approach, where, based on the likelihood values and the prior probabilities the model with the greater posterior probability is selected. With unknown prior probabilities, one cannot calculate posterior probabilities, so the

posterior probabilities must be approximated. Schwarz (1978) provides a simple and accurate approximation for this criterion. This method assumes equal prior probabilities for the candidate models. The Schwarz criterion (SC) includes an adjustment for the degrees of freedom that depends on both the number of parameters and the sample size.

Equation 13

$$SC = log[L(x|\Theta)] - klog(\sqrt{N}\ )$$

where $L(x|\Theta)$ is the value of the model's maximized likelihood function, N is the sample size and k is the number of independent parameters in the model.

The model that maximizes the SC value provides the best fit, since it is the model that has the highest likelihood value while controlling for the number of parameters in the model and the sample size. For large samples these approximations are good, but for arbitrary finite samples unless the exact form of the prior probability distribution of the parameter vector Θ is known it is not possible to check the accuracy of the SC approximation. Prior research conducts simulations, based on known parameters. They indicate that the Schwarz criteria accurately approximates true measure of fit for daily changes in exchange rates.

RESULTS AND CONCLUSION

Table 1 reports the summary statistics for the sample data. For most currencies, the means for the three five year subperiods shift. On the other hand, variances appear relatively stable for all four currencies.

Table 1 also reports test statistics for autocorrelation and heteroscedasticity. With three exceptions, the Durbin Watson test statistics indicate no autocorrelation at the .10 significance level. The three exceptions are the British pound, Canadian dollar and the German mark in the 3rd subperiod (Jan 88 - Dec 92). Lagrange multiplier (LM) test and the Portmanteua Q-test, test statistics for heteroscedasticity, are also reported. The LM and Q statistics are computed from the residuals of OLS, assuming a white-noise null hypothesis. The results on both LM statistics and Q statistics indicate significant first-order heteroscedasticity for the overall sample for all currencies (Jan 78 - Dec 92). In the first subperiod (Jan 78 - Dec 82), only Japan had an insignificant value for LM and Q. In the second subperiod (Jan 83 - Dec 87) again, all currencies have significant test statistics for first-order heteroscedasticity. For the last subperiod (Jan 88 - Dec 92) only the British pound has insignificant test statistics for heteroscedasticity.

Given significant heteroscedasticity the dependent-type GARCH models that can account for the changing variances would presumably provide a better fit than the independent-type models. Given a constant variance, the independent models (the mixed jump diffusion model or the discrete mixture of normals model) should fit the data better.

Tables 2 and 3 present each models daily maximum likelihood values, and Schwarz criteria; Table 4 summarizes the results. For the twelve five year subperiods, ten subperiods have p-values < .1, in the test for significant heteroscedasticity. GARCH type models are superior to the independent models in only 5 of the ten cases. For four of the five remaining significant cases the mixed jump diffusion model has the best fit. The mixture of 2 normals also dominates all GARCH models in these instances. For the overall period all four currencies have significant heteroscedasticity, yet two of the four currencies have outcomes that are best described by an independent model. Thus, although the data has significant heteroscedasticity in most cases, half the time, exchange rate changes are best fit by models that assume the outcomes are independent. For the two cases where the variance appears stable, a mixed jump diffusion model and a mixture of two normals model provide best fits for the data.

An interesting result occurs with the two cases where variance is stable, British Jan 88 - Dec 92, and Japanese Jan 78 - Dec 82 subperiods. Casual empiricism might suggest that the elimination of heteroscedasticity is due to GARCH-type models being used in practice, making the market more efficient and removing the heteroscedasticity. The problem with this line of reasoning is that if the use of the GARCH models removed the first-order heteroscedasticity, the parameters of the various GARCH(1,1) models should no longer be significant in these periods. All parameters except the conditional mean in the GARCH(1,1)-M model are found to be significant in periods where there is no first-order heteroscedasticity.

A question for future research is what are the GARCH models first-order parameters explaining if there is no first-order heteroscedasticity? They may be picking up higher order heteroscedasticity, if that is the case the models used here are misspecified. It could be argued that GARCH effects do exist in the data, but outliers are causing the tests for first order heteroscedasticity to be insignificant. The question then becomes why do the outliers play a more

significant role in the tests for first order heteroscedasticity than they do in the GARCH modeling process? It could be caused by differing assumptions between the two tests and the models. The original GARCH model assumes a normal distribution with conditional variance that changes over time. The LM and Q statistics are computed from OLS residuals assuming the disturbances are white noise. The Q and LM statistics have an approximate X^2 distribution under the white noise hypothesis. The X^2 distribution is skewed positively but as the degrees of freedom increases it approaches the shape of a normal distribution. Since our degrees of freedom in both tests are over 1,000, we believe that it is not the assumptions that are causing the differences found between the tests for first-order heteroscedasticity and the GARCH(1,1) type models.

Until these questions are answered the movement to more complex models, is premature.

Table 4 shows that neither the type of model (dependent or independent) nor the forms of the model type persist from one five year period to the next. The only exception being the Japanese yen which consistently has independent models fit the best. Table 4 also indicates there is no consistency as to model type or form across the currencies within any subperiod. Again, the independent form is the only exception in the Jan 88 - Dec 92 subperiod.

In the most recent subperiod (Jan 88 -Dec 92) tests show that for three of the four currencies first-order heteroscedasticity becomes less pronounced, insignificant for one of the three. It will be interesting to see if this trend continues in the future.

In summary, our results indicate that dependent GARCH models do not dominate the independent models. Since independence is still indicated, the movement of current research, using daily data, to purely dependent models is inappropriate. When heteroscedasticity is present in the data the GARCH type models are superior only half of the time when compared to the mixed jump diffusion model or a mixture of two normals model. Much of the existing research on the distribution of exchange rates suggests that high frequency data are not independent and identically distributed. However, we conclude that models which assume independence should not be overlooked since existing models of dependence do not dominate the alternatives which assume independence.

Future research should focus not on the search for the perfect GARCH model, but attempt to develop a model that incorporates the pronounced volatility clustering found in exchange rate price series and the independent behavior that obviously exists in some of the data.

TABLE 1
Summary Statistics for Three 5-year Subperiods and the Overall Period

Jan 78 - Dec 82	BP	CD	GM	JY
Mean	-.000152	-.000098	-.000113	.000012
Variance	.000046	.000008	.000052	.000058
Durbin-Watson(DW)	2.05	2.11	2.10	2.07
Prob < DW	.792	.978	.958	.907
Q	15.47*	186.19*	7.25*	.98
Prob > Q	.0001	.0001	.0071	.3229
LM	14.14*	185.73*	6.91*	.86
Prob > LM	.0002	.0001	.0086	.3493
Jan 83 - Dec 87	**BP**	**CD**	**GM**	**JY**
Mean	.000121	-.000041	.000331	.000525
Variance	.000074	.000008	.000066	.000043
Durbin-Watson(DW)	2.11	2.10	2.16	2.10
Prob < DW	.975	.954	.997	.958
Q	206.90*	201.61*	19.77*	26.74*
Prob > Q	.0001	.0001	.0001	.0001
LM	206.49*	201.14*	19.75*	26.73*
Prob > LM	.0001	.0001	.0001	.0001

TABLE 1
Summary Statistics for Three 5-year Subperiods and the Overall Period
(Cont'd)

Jan 88 - Dec 92	BP	CD	GM	JY
Mean	-.000173	.000020	-.000022	-.000024
Variance	.000059	.000007	.000056	.000043
Durbin-Watson(DW)	1.88*	1.86*	1.92*	1.95
Prob < DW	.013	.006	.087	.166
Q	.5162	11.46*	3.73*	3.38*
Prob > Q	.4725	.0007	.0535	.0658
LM	.5190	11.43*	3.70*	3.31*
Prob > LM	.4713	.0007	.0544	.0587
Jan 78 - Dec 92	**BP**	**CD**	**GM**	**JY**
Mean	-.000070	-.000039	.000064	.000169
Variance	.000059	.000007	.000058	.000048
Durbin-Watson(DW)	2.02	2.03	2.06	2.04
Prob < DW	.686	.791	.973	.899
Q	363.02*	445.08*	29.79*	23.40*
Prob > Q	.0001	.0001	.0001	.0001
LM	360.95*	444.70*	29.18*	23.09*
Prob > LM	.0001	.0001	.0001	.0001

BP: British Pound, CD: Canadian Dollar, GM: German Mark, JY: Japanese Yen
*significant for alpha < .1

ENDNOTES

1. They model the independence in the form of heterogenous information arrivals.

2. Using a t-distribution in place of the assumed normal distribution did not change the conclusions of this study.

3. Baillie and Bollerslev (1989) have also examined a GARCH-t model which uses a t-distribution in place of a normal distribution.

4. The ARCH-M model, introduced by Engel, Liben, and Robins (1987), allows the mean to shift, but as a function of the conditional variance process.

REFERENCES

1. Akgiray, V., and Booth, G., "Stock Price Processes With Discontinuous Time Paths: An Empirical Examination," *Financial Review*, 1986, pp. 163-184.

2. Akgiray, V., and Booth, G., "Mixed Diffusion-Jump Process Modeling Of Exchange Rate Movements," *Review of Economics and Statistics*, November 1988, pp. 631-637.

3. Anderson, T., Bollerslev, T., "Heterogeneous Information Arrivals And Return Volatility Dynamics: Uncovering the Long-Run In High Frequency Returns," *Journal of Finance*, 1997, pp. 975-1005.

4. Baillie, R., Bollerslev, T., "The Message In Daily Exchange Rates: A Conditional Variance Tale," *Journal Business and Economic Statistics*, 1989, pp. 297-305.

TABLE 2
Maximum Likelihood Results (Schwarz Criteria)
Daily Exchange Rate Data
Subperiod 1 and 2

Jan 78 - Dec 82	BP	CD	GM	JY
GARCH(1,1)	4561.52 (4547.25)*	5695.85 (5681.58)*	4533.58 (4519.31)	4361.90 (4347.63)
GARCH-M(1,1)	4564.57 (4546.73)	5698.96 (5681.12)	4533.87 (4516.03)	4362.85 (4345.01)
EGARCH(1,1)	4558.71 (4540.87)	5692.10 (5674.26)	4538.33 (4520.49)*	4364.31 (4346.47)
IGARCH(1,1)	4549.13 (4534.86)	5681.19 (5666.97)	4530.55 (4516.28)	4347.29 (4333.02)
Mixed Jump	4556.03 (4538.19)	5684.43 (5666.59)	4510.11 (4492.27)	4385.12 (4367.28)
Mixture of 2 Normals	4561.17 (4543.33)	5693.28 (5675.44)	4507.15 (4489.31)	4387.41 (4369.57)*
Jan 83 - Dec 87	**BP**	**CD**	**GM**	**JY**
GARCH(1,1)	4276.18 (4261.91)	5857.89 (5843.62)	4315.55 (4301.28)*	4573.53 (4559.26)
GARCH-M(1,1)	4276.77 (4258.93)	5860.45 (5842.61)	4318.21 (4300.37)	4576.49 (4558.65)
EGARCH(1,1)	4282.26 (4264.42)	5869.26 (5851.42)*	4317.65 (4299.81)	4578.44 (4560.60)
IGARCH(1,1)	4272.60 (4258.33)	5857.68 (5843.41)	4312.32 (4298.05)	4558.99 (4544.72)
Mixed Jump	4311.71 (4293.87)*	5832.77 (5814.93)	4292.89 (4275.05)	4622.58 (4604.74)*
Mixture of 2 Normals	4300.87 (4283.03)	5771.26 (5753.42)	4301.00 (4283.16)	4620.81 (4602.97)

BP: British Pound, CD: Canadian Dollar, GM: German Mark, JY: Japanese Yen
Table shows the log of the likelihood function and the Schwarz criteria()
*Largest Schwarz criteria for the period or subperiod

5. Baillie, R., Bollerslev, T., and Mikkelsen, H., "Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 1996, pp. 3-30.

6. Bollerslev, T., "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 1986, pp. 307-327.

7. Bollerslev, T., Chou, R., and Kroner, K., "ARCH Modeling In Finance," *Journal of Econometrics*, 1992, pp. 5-59.

8. Bollerslev, T., and Ghysels, E., "Periodic Autoregressive Conditional Heteroscedasticity," *Journal of Business and Economic Statistics*, 1996, pp. 139-151.

TABLE 3
Maximum Likelihood Results (Schwarz Criteria)
Daily Exchange Rate Data
Subperiod 2 and Overall

Jan 88 - Dec 92	BP	CD	GM	JY
GARCH(1,1)	4540.85 (4526.51)	5965.40 (5951.06)	4557.82 (4543.48)	4722.38 (4708.04)
GARCH-M(1,1)	4542.80 (4524.88)	5966.05 (5948.13)	4558.04 (4540.12)	4722.38 (4704.46)
EGARCH(1,1)	4547.98 (4530.06)	5967.29 (5949.37)	4565.40 (4547.48)	4722.39 (4704.47)
IGARCH(1,1)	4533.69 (4519.35)	5963.10 (5948.76)	4551.78 (4537.44)	4705.22 (4690.88)
Mixed Jump	4570.67 (4552.74)	5987.26 (5969.34)*	4568.65 (4550.72)	4773.83 (4755.90)*
Mixture of 2 Normals	4579.13 (4561.20)*	5985.30 (5967.37)	4569.88 (4551.95)*	4766.85 (4748.92)
Jan 78 - Dec 92	**BP**	**CD**	**GM**	**JY**
GARCH(1,1)	13383.91 (13367.42)	17502.45 (17485.96)	13412.46 (13395.97)	13662.28 (13645.79)
GARCH-M(1,1)	13384.81 (13364.19)	17503.24 (17482.62)	13412.68 (13392.06)	13644.25 (13643.63)
EGARCH(1,1)	13386.81 (13366.19)	17517.60 (17496.98)*	13425.68 (13405.06)*	13665.50 (13644.88)
IGARCH(1,1)	13386.98 (13370.49)	17500.30 (17483.81)	13403.38 (13386.89)	13623.32 (13606.83)
Mixed Jump	13430.62 (13410.01)*	17460.33 (17439.71)	13368.73 (13346.11)	13768.66 (13748.04)*
Mixture of 2 Normals	13420.59 (13399.97)	17464.86 (17444.24)	13364.79 (13344.17)	13757.14 (13736.52)

BP: British Pound, CD: Canadian Dollar, GM: German Mark, JY: Japanese Yen
Table shows the log of the likelihood function and the Schwarz criteria()
*Largest Schwarz criteria for the period or subperiod

9.   Boothe, P., and Glassman, D., "The Statistical Distribution Of Exchange Rates," *Journal of International Economics*, May 1987, pp. 297-319.

10. Engle, R., "Autoregressive Conditional Heteroscedasticity With Estimates Of The Variance Of United Kingdom Inflation," *Econometrica*, 1982, pp. 987-1007

11. Engle, R., Liben, D., and Robins, R., "Estimating Time Varying Risk Premia In The Term Structure: The ARCH-M Model," *Econometrica*, 1987, pp. 391-407.

12. Fujihara, R., and Park, K., "The Probability Distribution Of Futures Prices In The Foreign Exchange Market: A Comparison Of Candidate Processes," *Journal of Futures Markets*, 1990, pp. 623-641.

TABLE 4
Association of Significant Autocorrelation;
Significant Heteroscedasticity, and Best-Fit Model Types

	Number of Currencies With Significant (alpha < .1)	
Period	First Order Autocorrelation	First Order Heteroscedasticity
Jan 78-Dec 82	None of 4	3 of 4
Jan 83-Dec 87	None of 4	4 of 4
Jan 88-Dec 92	3 of 4	3 of 4
Jan 78-Dec 92	None of 4	4 of 4

Best-fit-to Currency

Period	BP	CD	GM	JY
Jan 78-Dec 82	GARCH(1,1)*	GARCH(1,1)*	EGARCH(1,1)*	Mixnormals
Jan 83-Dec 87	Mixjump*	EGARCH(1,1)*	GARCH(1,1)*	Mixjump*
Jan 88-Dec 92	Mixnormals#	Mixjump#*	Mixnormals#*	Mixjump*
Jan 78-Dec 92	Mixjump*	EGARCH(1,1)*	EGARCH(1,1)*	Mixjump*

BP: British Pound, CD: Canadian Dollar, GM: German Mark, JY: Japanese Yen
*period with significant 1st order heteroscedasticity
#period with significant 1st order autocorrelation

13. Hsieh, D., "The Statistical Properties Of Daily Exchange Rates: 1974-1983," *Journal of International Economics*, 1988, pp. 129-145.

14. Kon, S., "Models of Stock Returns - A Comparison," *Journal of Finance*, 1984, pp. 147-165.

15. Kugler, P., and Lenz, C., "Chaos, ARCH and the Foreign Exchange Market: Empirical Results From Weekly Data," Unpublished manuscript, Volkswirtschaftliches Institut, Zurich, 1990.

16. McCurdy, T., and Morgan, I., "Testing the Martingale Hypothesis In Deutsche Mark Futures With Models Specifying The Form Of The Heteroscedasticity," *Journal of Applied Econometrics* 3, 1988, pp. 187-202.

17. McFarland, J., Pettit, R., and Sung, S., "The Distribution Of Foreign Exchange Price Changes: Trading Day Effects And Risk Measurement," *Journal of Finance*, June 1982, pp. 693-715.

18. Merton, R., "Option Pricing When Underlying Stock Returns Are Discontinuous," *Journal of Financial Economics*, 1976, pp. 125-144.

19. Nelson, D., "Conditional Heteroscedasticity In Asset Returns: A New Approach," *Econometrica*, 1991, pp. 347-370.

20. Schwarz, G., "Estimating the Dimension of a Model," *Annals of Statistics*, 1978, pp. 461-464.

21. Taylor, S., "Modeling Financial Time Series," *Wiley*, New York, NY, 1986.

22. Tucker, A., and Pond, L., "The Probability Distribution Of Foreign Exchange Price Changes: Tests Of Candidate Processes," *Review of Economics and Statistics*, November 1988, pp. 638-647.