# THE IMPACT OF A PRICE CUT ON NET INCOME AND PROFIT MARGIN 

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#### Abstract

A reduction in the price of a product may improve the profit for the stockholders due to more demand for the same product. In this paper, three guidelines are offered to practitioners who are considering a price cut. First, a price reduction is justified only if the price elasticity of demand is less than a benchmark value, which is derived in this article. Second, there exists the optimum level of price cut, which maximizes the stockholders' wealth. Third, a declined profit margin after a price cut does not necessarily impair the stockholders' wealth.


## INTRODUCTION

Companies often times cut the price of goods in the hope that a small reduction in price may lead to a substantial increase in the quantity demanded, and consequently a sizable gain in profit. Whether or not the price reduction may boost the profit depends upon the price elasticity of demand of the product under consideration. In this paper, the condition of the price elasticity, which increases the profit after a price cut, is derived to help managers improve the wealth of the stockholders.

In economics textbooks, the profit-maximizing optimum price policy depends upon the effect of the price elasticity on the marginal revenue and marginal cost. See Mansfield (1996), and Hirschey and Pappas (1996). The problem is that the relationship between the price elasticity and the marginal cost is not precisely assessed, since usually a hypothetical marginal cost curve is assumed in economics literature, e.g., a monotone increasing cost function in quantity demanded. In this paper, the order of the income statement entries is followed to evaluate how the price elasticity may affect the revenue and cost, because this procedure is actually taken in the real world to measure the profit for the stockholders.

Additionally this paper addresses the impact of a price reduction on the (net) profit margin. The general consensus is that a low profit margin is an indication of operating inefficiency resulting from low sales, high costs, or both (Brigham, 1995; Brigham and Gapenski, 1997; and Pinches, 1996), and that a high profit margin serves the stockholders (Lazere, 1996; Sullivan, 1996; and Esquivel, 1996). However, a low profit margin is not necessarily bad for the stockholders, because, even if a reduction in price may result in a low profit margin, the net income may increase due to the increased demand for the product. The direction and magnitude of the change in profit margin also depends upon the price elasticity of the product.

The purpose of this paper is two-fold. First, it derives the condition, under which the net income rises after a price cut, to help practitioners decide whether or not to reduce the price. Second, it identifies the condition, under which a price cut increases the profit margin, to convince the readers that a low profit margin does not always hurt the stockholders. Additionally, the optimum price cut, that maximizes stockholders' wealth, is also shown. Numerical examples are provided to verify the results of the study.

## PROFIT MAXIMIZATION VERSUS WEALTH MAXIMIZATION

For a public corporation, the profit maximization criterion supported by economists is inappropriate as the firm's objective, because it ignores the time value of money and the degree of risk. The kinds of risk that matters include, but are not limited to, business, financial, and operating risks. The degree of business risk depends on the line of

[^0]business that the company is engaged in. For example, the business risk of an airline company is higher than that of a grocery chain. Financial risk increases as the firm uses more long-term debt, that produces fixed financial commitments. The operating risk of a firm is measured by the types of capital assets that the company uses. A labor intensive capital asset has a low operating risk; a capital intensive capital asset creates more operating risk.

Wealth maximization is more appropriate for the stockholders of a public corporation, because both the time value of money and the degree of risk are accounted for in the efficient stock market. In this paper, however, net income maximization is chosen in place of wealth maximization for three valid reasons. First, net income, which is the measure of profit for the stockholders, is the main interest of financial practitioners as in Greenwald (1996), and Hensley (1997). Second, net income after a price reduction is easier to project, thereby providing an analytical edge and expositional conveniences. Third, net income maximization is usually consistent with wealth maximization, because the three kinds of risks mentioned earlier do not change drastically. ${ }^{1}$

## PRICE ELASTICITY

To measure how a price cut may change the quantity demanded, and consequently net income and profit margin, the price elasticity of demand of the following equation is examined in this section:

## Equation 1

$$
\eta=-\left(\frac{\partial Q}{Q} / \frac{\partial P}{P}\right)
$$

where $\eta$ is the price elasticity, $Q$ is the original quantity, and $P$ is the original price. The numerator of this equation is the percentage increase in quantity; the denominator is the percentage cut in the price.

## THE IMPACT OF THE PRICE ELASTICITY ON NET INCOME AND PROFIT MARGIN

To determine how the price elasticity may affect the net income, the net income (NI) before the price cut is measured first as follows:

## Equation 2

$$
N 1=(1-\tau)[P Q(1-V)-F-I]
$$

where $\tau$ is the corporate tax rate, $V$ is the variable operating cost as a percentage of the original price, $F$ is the fixed operating cost, and I is the fixed interest expense. Then, the new level of net income after the price reduction $\left(N I^{N}\right)$ is given by:

## Equation 3

$$
\begin{aligned}
N I^{N} & =(1-\tau)[P(1-x) Q(1-\eta x)-V P Q(1-\eta x)-F-I] \\
& =(1-\tau)[P Q(1-\eta x)(1-x-V)-F-I]
\end{aligned}
$$

See Appendix A for the derivation of equation (3). This new net income after the price cut is not necessarily larger than the original amount of net income of equation (2).

For wealth-maximizing managers, Appendix B shows the condition, under which the net income increases after a price cut, as follows:

Equation 4

$$
\eta<-\frac{1}{1-x-V}
$$

where $x$ is the fraction of the price reduction from the original price. ${ }^{2}$ If the above condition is met, the net income will increase after the price cut; otherwise it will be the same as or less than the original net income. Equation (4) offers three implications. First, as the demand of the product gets more elastic, it is more likely that the net income
would increase after the price reduction. Second, the chance of higher net income decreases, when the company cuts the price more. Third, the probability of higher net income also decreases, if the variable operating cost as a percentage of the original price increases.

Taking advantage of equation (3), the optimum level of price cut $\left(x^{*}\right)$, which maximizes the wealth of the stockholders, is derived as follows in Appendix C:

Equation 5

$$
x^{*}=\frac{l+\eta-\eta V}{2 \eta}
$$

If the price is cut more or less than $(1+\eta-\eta V) / 2 \eta$, the net income will be below the maximum possible amount. Hence, the wealth-maximizing manager should cut the price to the level indicated by the above condition.

After a price cut, the profit margin may increase or decrease depending upon the price elasticity and other factors. To show the profit margin increasing condition, the profit margin ( m ) is examined here, which is defined as:

Equation 6

$$
m=\frac{N I}{S a l e s}=\frac{N I}{P Q}
$$

It is generally agreed among textbook writers that the higher this margin, the better. See Block and Hirt (1994, pp. 55-56), Gitman (1997, p. 136), and Van Horne and Wachowicz (1995, pp. 140). Also agreed is that a low profit margin is bad for the stockholders, because it indicates high costs from inefficient operations and heavy use of debt (Brigham; 1995, p. 79), or low sales, high costs, or both (Brigham and Gapenski, 1997, pp. 52-53; and Weston and Brigham, 1993, p.57). Keown, Scott, Martin, and Petty (1996, footnote 9 on p.100) take a more in-depth look to conclude that the profit margin is affected by the operating and financing activities of the company.

However, a low profit margin is not necessarily bad for the stockholders. The reason is that, although the reduced price may decrease the profit margin, the net income may improve after a price cut due to the more than proportionately increased quantity. The following condition is derived by Appendix D , under which the profit margin increases after a price cut by $x$ :

Equation 7

$$
\frac{P Q(1-\eta x)(V)}{F+I}<-1-\eta+\eta x
$$

The numerator on the left side of the above inequality is the total variable cost after the cut, and its denominator is the total fixed cost before and after the price reduction.

The profit margin will increase after a price cut if this ratio is less than the benchmark value, which is $-1-\eta+\eta x$. If this ratio is greater than that, the profit margin will decline even if the net income increases. The implication is that managers should go by the net income after the price cut because it is the true representative measure of the stockholders' wealth. The profit margin should not be considered in the decision-making process of a price reduction.

## NUMERICAL EXAMPLES

In this section, numerical illustrations are provided to support the results of the study, using the hypothetical data in Table 1.

## The Net Income Improving Condition

Using the data and equation (4), the breakeven $\eta$ is computed as follows, at which the NI of the company does not change before and after a 2.5 percent price cut from $\$ 2,000$ to $\$ 1,950$;

$$
\text { Breakeven } \eta=\frac{1}{1-2.5 \%-40 \%}=\frac{1}{57.5 \%}=1.73913
$$

## TABLE 1 Hypothetical Data

| Original quantity: | 3,000 units |
| :--- | :--- |
| Original price: | $\$ 2,000$ |
| Variable operating cost per unit: | 40 percent of the original price or $\$ 800$ |
| Fixed operating cost: | $\$ 1,000,000$ |
| Fixed interest expense: | $\$ 800,000$ |
| Tax rate: | 40 percent |

Table 2 shows the income statements before and after the 2.5 percent price cut at different levels of $\eta$ ranging from -1 to -2.5 .

TABLE 2

## The Net Income Improving Condition

| Original Quantity | 3,000 |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Original Price | $\$ 2,000$ |  |  |  |  |
| New Price | $\$ 1,950$ | -1 | -1.5 | -1.73913 | -2 |
| Price Elasticity | 3,075 | 3,113 | 3,130 | 3,150 | 3,188 |
| New Quantity |  |  |  |  |  |

INCOME STATEMENTS

|  | Original | New | New | New | New | New |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Revenue |  |  |  |  |  |  |
| Less Variable | $\$ 6,000,000$ | $\$ 5,996,250$ | $\$ 6,070,350$ | $\$ 6,103,500$ | $\$ 6,142,500$ | $\$ 6,216,600$ |
| Operating Cost | $\$ 2,400,000$ | $\$ 2,460,000$ | $\$ 2,490,400$ | $\$ 2,504,000$ | $\$ 2,520,000$ | $\$ 2,550,400$ |
| Less Fixed |  |  |  |  |  |  |
| Operating Cost | $\$ 1,000,000$ | $\$ 1,000,000$ | $\$ 1,000,000$ | $\$ 1,000,000$ | $\$ 1,000,000$ | $\$ 1,000,000$ |
| Earnings Before |  |  |  |  |  |  |
| Interest and Taxes | $\$ 2,600,000$ | $\$ 2,536,250$ | $\$ 2,579,950$ | $\$ 2,599,500$ | $\$ 2,622,500$ | $\$ 2,666,200$ |
| Less Fixed | $\$ 800,000$ | $\$ 800,000$ | $\$ 800,000$ | $\$ 800,000$ | $\$ 800,000$ | $\$ 800,000$ |
| Interest Expense | $\$ 1,800,000$ | $\$ 1,736,250$ | $\$ 1,779,950$ | $\$ 1,799,500$ | $\$ 1,822,500$ | $\$ 1,866,200$ |
| Earnings Before Taxes | $\$ 720,000$ | $\$ 694,500$ | $\$ 711,980$ | $\$ 719,800$ | $\$ 729,000$ | $\$ 746,480$ |
| Less Taxes @ 40\% | $\$ 1,080,000$ | $\$ 1,041,750$ | $\$ 1,067,970$ | $\$ 1,079,700$ | $\$ 1,093,500$ | $\$ 1,119,720$ |
| Net Income | $18.00 \%$ | $17.37 \%$ | $17.59 \%$ | $17.69 \%$ | $17.80 \%$ | $18.01 \%$ |
| Profit Margin | Loss | Loss | Breakeven | Gain | Gain |  |
| Gain or Loss |  |  |  |  |  |  |

Notes: New Quantity = Original Quantity $+\eta \times(($ New Price - Original Price) $/$ Original Price $) \times$ Original Quantity.
Revenue $=$ Price $\times$ Quantity
Variable Operating Cost $=40 \% \times$ Original Price $\times$ Quantity
Profit Margin $=$ Net Income $/$ Revenue
*The difference is due to rounding errors.

The stockholders will be worse off, if the price elasticity is greater than -1.73913; if it is less than that its net income will improve; when the price elasticity is the same as this benchmark value, the net income will not change after the price reduction.

## The Optimum Price Cut

The optimum level of price cut, which maximizes the net income, is calculated as follows, using the data and equation (5), for a product with $\eta=-2$ :

$$
x^{*}=\frac{1+(-2)-(-2)(40 \%)}{2(-2)}=\frac{-.2}{-4}=5 \%
$$

Table 3 presents the net income at different levels of price cut ranging from 3 to 7 percent for a product with $\eta=$ -2 ; the Figure is the graphical presentation of Table 3.

TABLE 3
The Optimum Price Cut: Income Statement Approach

| Original Quantity | 3,000 |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Original Price | $\$ 2,000$ |  |  |  |  |
| Percent Cut | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ |
| New Price | $\$ 1,940$ | $\$ 1,920$ | $\$ 1,900$ | $\$ 1,880$ | $\$ 1,860$ |
| Price Elasticity | -2 |  |  |  |  |
| New Quantity | 3,180 | 3,240 | 3,300 | 3,360 | 3,420 |

## INCOME STATEMENTS

|  | Original | New 3\% Cut | $\begin{gathered} \text { New } \\ \mathbf{4 \% ~ C u t} \end{gathered}$ | $\begin{aligned} & \text { New } \\ & \text { 5\% Cut* } \end{aligned}$ | New $6 \%$ Cut | New <br> 7\% Cut |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Revenue | \$6,000,000 | \$6,169,200 | \$6,220,800 | \$6,270,000 | \$6,316,800 | \$6,361,200 |
| Less Variable |  |  |  |  |  |  |
| Operating Cost | \$2,400,000 | \$2,544,000 | \$2,592,000 | \$2,640,000 | \$2,688,000 | \$2,736,000 |
| Less Fixed |  |  |  |  |  |  |
| Operating Cost | \$1,000,000 | \$1,000,000 | \$1,000,000 | \$1,000,000 | \$1,000,000 | \$1,000,000 |
| Earnings Before <br> Interest and Taxes | \$2,600,000 | \$2,625,200 | \$2,628,800 | \$2,630,000 | \$2,628,800 | \$2,625,200 |
| Less Fixed Interest Expense | \$800,000 | \$800,000 | \$800,000 | \$800,000 | \$800,000 | \$800,000 |
| Earnings Before Taxes | \$1,800,000 | \$1,825,200 | \$1,828,800 | \$1,830,000 | \$1,828,800 | \$1,825,200 |
| Less Taxes @ 40\% | \$720,000 | \$730,080 | \$731,520 | \$732,000 | \$731,520 | \$730,080 |
| Net Income | \$1,080,000 | \$1,095,120 | \$1,097,280 | \$1,098,000 | \$1,097,280 | \$1,095,120 |
| Profit Margin | 18.00\% | 17.75\% | 17.64\% | 17.51\% | 17.37\% | 17.22\% |

Notes: New Price $=($ Original Price $) \times(1-$ Percentage Cut in Price $)$
New Quantity $=($ Original Quantity $)(1-\eta \times$ Percentage Cut in Price $)$
Revenue $=$ Price $\times$ Quantity
Variable Operating Cost $=40 \% \times$ Original Price $\times$ Quantity
Profit Margin $=$ Net Income $/$ Revenue
*Indicates the optimum level of price cut, that maximizes the net income.

FIGURE
The Optimum Price Cut:
A Graphical Presentation

$\mathrm{x}^{*}$ is the optimum level of price cut, at which the net income is maximized.

For a product with $\eta=-2$, the net income increases as the company cuts the price more up to 5 percent; after that point it decreases; hence, the optimum price cut is 5 percent.

## The Profit Margin Enhancing Condition

A product with $\eta=-2$ is investigated in this example with a $2.5 \%$ price cut from $\$ 2,000$ to $\$ 1,950$. As a result, the new quantity will increase from 3,000 to 3,150 . In this example, only the ratio of the total variable operating cost after the price cut to the total fixed cost before and after the cut (hereafter, the ratio) is allowed to change from 85 percent to 105 percent. Using equation (7) and the data, the breakeven point of this ratio, at which the profit margin is the same before and after the price cut, is calculated as follows:

$$
\text { Breakeven ratio }=-1-(-2)+(-2)(2.5 \%)=95 \%
$$

The original and new income statements at different levels of this ratio are calculated and reported in Table 4. The new income statements are made prior to the original counterparts in this Table. Based upon the new price and the new quantity, the revenue and the variable operating cost after the price cut are computed. The total fixed cost after the price cut, the sum of the fixed operating cost and the fixed interest expense, is then computed, using the total variable operating cost and the ratio. Thus calculated total fixed cost is then entered in the corresponding original income statements depending upon the ratio.

Table 4 shows that the net income has increased regardless of the level of the ratio, because the price elasticity is less than -1.73913 . When this ratio is less than 95 percent, the profit margin has also increased; if this ratio goes beyond 95 percent, then the profit margin declines although the net income has improved; when this ratio is exactly 95 percent, the profit margin is the same before and after the price cut at 9.47 percent. The foregoing observations clearly verify the validity of the breakeven ratio given by equation (7).

TABLE 4 The Profit Margin Enhancing Condition

| Original Quantity |  | 3,000 |  |  |
| :--- | ---: | ---: | ---: | :--- |
| Original Price | $\$ 2,000$ |  |  |  |
| New Price | $\$ 1,950$ |  |  |  |
| Price Elasticity | -2 |  |  |  |
| New Quantity | 3,150 |  |  |  |
| Ratio | $85 \%$ | $90 \%$ | $95 \%$ | $100 \%$ |

ORIGINAL INCOME STATEMENTS

| Revenue <br> Less Variable <br> Operating Cost | $\$ 6,000,000$ | $\$ 6,000,000$ | $\$ 6,000,000$ | $\$ 6,000,000$ | $\$ 6,000,000$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Less Total | $\$ 2,400,000$ | $\$ 2,400,000$ | $\$ 2,400,000$ | $\$ 2,400,000$ | $\$ 2,400,000$ |
| Fixed Cost | $\$ 2,964,706$ | $\$ 2,800,000$ | $\$ 2,652,632$ | $\$ 2,520,000$ | $\$ 2,400,000$ |
| Earnings Before Taxes | $\$ 635,294$ | $\$ 800,000$ | $\$ 947,368$ | $\$ 1,080,000$ | $\$ 1,200,000$ |
| Less Taxes @ 40\% | $\$ 254,118$ | $\$ 320,080$ | $\$ 378,947$ | $\$ 432,000$ | $\$ 480,000$ |
| Net Income | $\$ 381,176$ | $\$ 480,000$ | $\$ 568,421$ | $\$ 648,000$ | $\$ 720,000$ |
| Profit Margin | $6.35 \%$ | $8.00 \%$ | $9.47 \%$ | $10.80 \%$ | $12.00 \%$ |

NEW INCOME STATEMENTS

| Revenue | \$6,142,500 | \$6,142,500 | \$6,142,500 | \$6,142,500 | \$6,142,500 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Less Variable |  |  |  |  |  |
| Operating Cost | \$2,520,000 | \$2,520,000 | \$2,520,000 | \$2,520,000 | \$2,520,000 |
| Less Total |  |  |  |  |  |
| Fixed Cost | \$2,964,706 | \$2,800,000 | \$2,652,632 | \$2,520,000 | \$2,400,000 |
| Earnings Before Taxes | \$657,794 | \$822,500 | \$969,868 | \$1,102,500 | \$1,222,500 |
| Less Taxes @ 40\% | \$263,118 | \$329,000 | \$378,947 | \$441,000 | \$489,000 |
| Net Income | \$394,676 | \$493,500 | \$581,921 | \$661,500 | \$733,500 |
| Profit Margin | 6.43\% | 8.03\% | 9.47\% | 10.77\% | 11.94\% |
| Gain or Loss | Gain | Gain | Same | Loss | Loss |

Notes: New Quantity $=3,000+(-2)((\$ 1,950-\$ 2,000) / \$ 2,000)(3,000)=3,150$
Ratio is the ratio of the total variable cost after the price cut to the total fixed cost before (and after) the price cut. To comply with this ratio, the total variable cost and the total fixed cost after the price cut are computed first. Then the same total fixed cost is transferred to the original income statements.
Profit Margin $=$ Net Income $/$ Revenue

## SUMMARY AND CONCLUSION

The price elasticity of demand for a product holds significant implications for managers considering a price reduction in an effort to improve the net income for the stockholders. This paper derives a constraint on the price elasticity under which the net income would increase after a price cut. The optimum level of price reduction is also shown, which maximizes the wealth of the stockholders. In addition, this paper identifies the condition, under which the profit margin increases after a price cut, to contend that a low profit margin is not always bad for the stockholders. The implication of this condition is that, even if the profit margin decreases, a price cut is still worthwhile as long as the net income increases.

## APPENDIX A

The purpose of this Appendix is to show the new level of net income after a price cut. If the price of a product is reduced by a small fraction $x$, the new price $\left(P^{\vee}\right)$ is given by:

## Equation A-1

$$
P^{N}=P-\partial P=P(1-x)
$$

The above equation produces:

## Equation A-2

$$
x=\frac{\partial P}{P}
$$

Substituting equation (A-2) into equation (1),

## Equation A-3

$$
\eta=-\left(\frac{\partial Q}{Q} / x\right)
$$

By cross multiplication,

## Equation A-4

$$
\eta x=-\frac{\partial Q}{Q}
$$

By multiplying both sides with $-Q$, and switching sides,

## Equation A-5

$$
\partial Q=-\eta x Q
$$

Using (A-5), the increased level of quantity after the price cut $\left(Q^{N}\right)$ is given by:
Equation A-6

$$
Q^{N}=Q+\partial Q=Q-\eta x Q=Q(1-\eta x)
$$

Multiplying (A-1) with (A-6), the new revenue ( $R^{N}$ ) after the price cut becomes:

## Equation A-7

$$
R^{N}=P(1-x) Q(1-\eta x)
$$

Similarly, the new $\operatorname{cost}\left(C^{N}\right)$ after the price cut is calculated as:

## Equation A-8

$$
C^{N}=V P Q(1-\eta x)+F+I
$$

where the first term in its right side is the new variable operating cost. Putting (A-7) and (A-8) together, the new level of $N I$ or $N I^{N}$ is given by:
Equation A-9

$$
\begin{aligned}
N I^{N} & =(1-\tau)[P(1-x) Q(1-\eta x)-V P Q(1-\eta x)-F-I] \\
& =(1-\tau)[P Q(1-\eta x)(1-x-V)-F-I]
\end{aligned}
$$

## APPENDIX B

This Appendix shows the net income improving condition after a price is cut by a fraction of $x$. The change in net income after the price cut is calculated by subtracting NI in equation (2) from $N I^{N}$ of equation (3) as follows:

## Equation B-1

$$
\begin{aligned}
N I^{N}-N I & =(1-\tau)[P Q(1-\eta x)(1-x-V)-F-I]-(1-\tau)[P Q(1-V)-F-I] \\
& =(1-\tau) P Q[(1-\eta x)(1-x-V)-(1-V)]
\end{aligned}
$$

The net income will increase after the price cut, if $\left(N I^{N}-N I\right)$ given by the above equation is positive or equivalently:

## Equation B-2

$$
(1-\eta x)(1-x-V)-(1-V)>0
$$

The above inequality produces:

## Equation B-3

$$
1-x-V-\eta x+\eta x^{2}+\eta x V-1+V>0
$$

which reduces to:

## Equation B-4

$$
-x-\eta x+\eta x^{2}+\eta x V>0
$$

Dividing both sides by $x$ produces:

## Equation B-5

$$
-1-\eta+\eta x+\eta V>0
$$

By adding 1 to both sides,
Equation B-6

$$
-\eta+\eta x+\eta V>1
$$

Multiplying both sides with -1 generates:

## Equation B-7

$$
\eta-\eta x-\eta V<-1
$$

Then:

## Equation B-8

$$
\eta(1-x-V)<-1
$$

By dividing both sides by $(1-x-V)$,

## Equation B-9

$$
\eta<-\frac{1}{1-x-V}
$$

If the above condition is met, the net income increases when the company cuts the price by a fraction $x$.

## APPENDIX C

In this Appendix, the optimum level of price cut, which maximizes the net income for the stockholders, is derived. For that purpose, the first derivative of equation (3) is obtained as follows:

## Equation C-1

$$
\begin{aligned}
\frac{\partial N I^{N}}{\partial x} & =(1-\tau) P Q[(1-\eta x)(-1)+(-\eta)(1-x-V)] \\
& =(1-\tau) P Q[-1+\eta x-\eta+\eta x+\eta V]=(1-\tau) P Q(-1+2 \eta x-\eta+\eta V)
\end{aligned}
$$

Its second derivative is given by:

## Equation C-2

$$
\frac{\partial^{2} N I^{N}}{\partial x^{2}}=(1-\tau) P Q 2 \eta<0
$$

The above two relations indicate that there exists an interior solution to optimum $x$. To solve for this optimum, equation (C-1) is set to zero as follows:

## Equation C-3

$$
-1+2 \eta x^{*}-\eta+\eta V=0
$$

where $x^{*}$ is the optimum level of $x$. Hence,

## Equation C-4

$$
2 \eta x^{*}=1+\eta-\eta V
$$

By dividing both sides by $2 \eta$,

## Equation C-5

$$
x^{*}=\frac{1+\eta-\eta V}{2 \eta}
$$

## APPENDIX D

This Appendix shows the profit margin increasing condition after a price cut by a fraction of $x$. Using equations $(2)$ and (6), the original profit margin ( $m$ ) is given by:

## Equation D-1

$$
m=\frac{(1-\tau)[P Q(1-V)-F-I]}{P Q}
$$

Similarly, using equations (3), (A-1), and (A-6), the new profit margin $\left(m^{N}\right)$ is calculated as:

## Equation D-2

$$
m^{N}=\frac{(1-\tau)[P Q(1-\eta x)(1-x-V)-F-I]}{P(1-x) Q(1-\eta x)}
$$

The profit margin will increase after the price cut under the following condition:
Equation D-3

$$
m^{N}-m=\frac{(1-\tau)[P Q(1-\eta x)(1-x-V)-F-I]}{P(1-x) Q(1-\eta x)}-\frac{(1-\tau)(1-x)(1-\eta x)[P Q(1-V)-F-I]}{P(1-x) Q(1-\eta x)}>0
$$

The numerators $(N R)$ of the right side of the above relation add up to:

## Equation D-4

$$
\begin{aligned}
N R= & (1-\tau) P Q(1-\eta x)(1-x-V)-(1-\tau)(F+I)-(1-\tau)(1-x)(1-\eta x) P Q(1-V)+(1-\tau)(1-x)(1-\eta x)(F+I) \\
& =(1-\tau) P Q(1-\eta x)[1-x-V-(1-x)(1-V)]-(1-\tau)(F+I)[1-(1-x)(1-\eta x)]
\end{aligned}
$$

Therefore, $m^{N}-m$ is positive, if $N R$ is positive, or equivalently:
Equation D-5

$$
(1-\tau) P Q(1-\eta x)(-V x)-(1-\tau)(F+I)[1-(1-x)(1-\eta x)]>0
$$

By dividing both sides by $-(1-\tau) x$, and rearranging terms,
Equation D-6

$$
P Q(1-\eta x)(V)<\frac{-(F+I)[1-(1-x)(1-\eta x)]}{x}
$$

By dividing both sides by $F+I$,
Equation D-7

$$
\frac{P Q(1-\eta x)(V)}{F+I}<\frac{-1+(1-x)(1-\eta x)}{x}
$$

which reduces to:
Equation D-8

$$
\frac{P Q(1-\eta x)(V)}{F+I}<\frac{-1+1-\eta x-x+\eta x^{2}}{x}
$$

Hence, the profit margin increasing condition is given by:
Equation D-9

$$
\frac{P Q(1-\eta x)(V)}{F+I}<-1-\eta+\eta x
$$

## ENDNOTES

1. The three risks would remain the same, as long as the company stays in the same line of business, keeps the similar capital structure, and operates with the same types of capital assets.
2. The fraction $x$ should be a small number, maybe no greater than 5 percent, because $\eta$ is sensitive to large-scale changes.

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