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RATCHET OPTIONS

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Abstract

This paper develops a general framework for the evaluation of both regular and compound ratchets. The specific model analyzed is for call ratchets since these are the most commonly found in the insurance industry. The results are both intuitive and interesting. As the Cap Rate of the ratchet increases (decreases) both the value of the ratchet and the sensitivity of the ratchet increase (decreases). The fixed income piece of the total portfolio is more influential under a changing interest rate environment. The model is easily extended to price any European type ratchet structure. Future research should concentrate on American and path dependent type ratchets.

INTRODUCTION

Nowhere in insurance is the bind between mathematics and financial economics stronger than in the pricing and evaluation of contingent claims. The proliferation of contingent claim structures throughout the insurance industry over the past few years has been extraordinary. Many of these structures are easy to price while others are extremely difficult; to make matters worse both the simple and the complex structures have embedded regulatory features that increase the level of sophistication required to properly evaluate them. It may be that the organizations currently selling these products are letting the marketing dictate both the finance and actuarial properties of the securities.

The derivative structures most commonly issued include lookbacks, arithmetic asians, and ratchets. Lookbacks come in varying forms – fixed strike and floating strikes. Floating strike calls are sometimes referred to as "low water" calls since the final payout is based on the minimum price over the life of the option and the final price at expiration of the option. Similarly, fixed rate calls are referred to as "high water" calls since the final payment depends on the maximum price over the life of the option and a fixed strike. Lookback put options are described in an analogous fashion. If the sampling for the maximum or minimum price is continuous then these options have analytic solutions as shown in Goldman, Sosin, and Gatto (1979). When the sampling is discrete, however, we need to utilize a numerical procedure. Arithmetic asian options average the underlying over a period of the options life. This average is then used to determine the final payoff of the option. Usually the averaging period is the last few months of the options life. No known analytic solution to these types of options exist whether sampling is continuous or discrete. Kemna and Vorst (1990) show how they are evaluated using a numerical approach. The ratchet option comes in two forms: the regular ratchet and the compound ratchet. The properties of these structures will be explained in detail in the next section. A discussion of a simple version of the regular ratchet can be found in Howard (1995). The purpose of this paper is to present a complete evaluation of both ratchet structures.

Pricing contingent claims or derivatives involves the determination of whether an analytical solution exists. This determination is not straightforward since many of the structures can be engineered as packages of more simple securities. The financial engineer must be creative and thorough since an analytic solution offers some advantages over the alternative numerical approaches. However, the analytic solution must be workable. Many of the analytic solutions available involve complex integrals - which then must be solved using a numerical approach. Clearly, this defeats the purpose of a closed form expression. Other structures, which seem non-markovian (with no reasonable analytic solution), turn out to be interesting packages of more simple securities. Finally, if the structure involves a discrete sampling dependency, as in many lookback options, then we must use a numerical approach even if we have an analytic solution with continuous sampling.

When no analytical solution exists we must determine the best numerical approach. There are three basic numerical approaches: finite differences, binomial (or trinomial), and Monte Carlo. Each approach has its advantages and disadvantages. Monte Carlo (MC) is mostly used when there is a path dependency property required

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to determine the final payoff. Boyle's (1977) seminal work was the first to show how MC can be used to price contingent claims. Kemna and Vorst (1990) use this approach to evaluate the asian option. Discrete lookbacks can also be evaluated using this approach. The finite difference method (FDM) requires a partial differential equation (PDE) and the accompanying boundary conditions in order to be implemented. FDM has many attractive features but requires an in depth understanding of PDE's. Buetow and Sochacki (1997) offer an excellent literature review of the FDM and how it has been used in derivative pricing. The binomial (and trinomial) methodology is the arguably the simplest of the three but requires recombination in order for meaningful implementation. Cox, Ross, and Rubinstein (1979) were the first to introduce this methodology to the derivative pricing landscape. Hull and White (1993) show how to use the binomial approach for path dependent options. All three are used extensively within derivative pricing; the best approach is to use more than one-method and compare results in order to ensure accuracy.

The paper is outlined as follows. Section II explains the properties of the ratchet structures. Section III develops the pricing equations and explains the characteristics of the ratchets. Section IV analyzes the price sensitivities of the structures. Section V concludes the paper.

THE RATCHET STRUCTURES

As mentioned in the previous section the ratchet structures have two basic forms: regular and compound. The regular ratchet consists of a sequence of (annual) resetting at-the-money contingent claims (CC).[1] The term ratchet is used to describe the resetting property – the moneyness of the CC is ratcheted up or down to at-the-money depending on how the underlying behaved during the previous year. The cash flows from the annual CC's are paid out as each annual CC expires and are therefore locked in. The cash flows are determined using a fixed notional amount. The maximum return on each CC is determined by a cap rate (CR). The effective CR is a function of the participation rate (PR) which is determined by the insurance company. The PR is the amount that the owner of the ratchet will *participate* in increases in the underlying. For example, if the PR is one then the owner will realize the same return as the underlying during the life of each CC.

The compound ratchet is similar except that there are no interim distributions of cash; instead of paying out the cash flows, the gains are "rolled" into the next CC thus increasing the number of CC's on a fixed notional. The number of CC's is based upon the returns of the underlying during each resetting period. Additionally, negative returns are *not* compounded so the compounding property actually adds another contingency to the structure. Consequently, the number of options in the final year will be determined by the total positive annual returns throughout the life of the compound ratchet. We show that this path dependency does not alter the markovian properties of the underlying and that the absence of interim cash flows will cheapen the compound ratchet relative to the regular ratchet. As with the regular ratchet the choice of the CR will have a significant effect on the value of the compound ratchet and is often used to obtain a PR of 1. The type of CC composing the ratchet structure can vary but most ratchets consist of European call options. Since this is the case we develop our model using European calls. However, our model is easily adapted to *any* type of European CC.

Since the compound ratchet and the regular ratchet have similar properties their values can be expressed by a single equation. In fact, the regular ratchet will be shown to be a special case of the compound ratchet. Both structures have the property that the amount of underlying asset (notional or principal) that is controlled by the contingent claim is constant throughout the life of the ratchet[2] (denoted by T years) and that the returns are locked in at the end of each resetting period (year). This is the usual characteristic of these packages. Normally, the underlying asset is a specific equity index. The indices currently used range from the S&P 500 index to proprietary multi country indices. Fortunately, the chosen index in no way alters the derivation.

Our model is derived under the same one factor model used to derive most accepted equity CC's. The assumptions are the same as those from the original Black and Scholes (1973) derivation. However, we will allow the current interest rate and volatility term structures to be incorporated into the individual CC's composing the ratchet structures. We feel that this is more accurate than assuming flat term structures.

Some may argue that a two-factor model consisting of stochastic interest rates and equity indices is more accurate. Equilibrium two-factor models are easily evaluated as in Buetow and Sochacki (1995) but they have no basis for comparison. Furthermore, these models must include an accurate estimate quantifying the relationship between interest rates and equity indices. As a result most derivative dealers do not use two factor models to evaluate structures that are primarily based on an equity index. It is for this reason that a one-factor model taking the two term structures, as inputs is the most accurate and widely used. The model introduced here has been used successfully to compare prices with those quoted by many large US investment firms.

THE PRICING EQUATIONS

To simplify the derivation of the valuation model we define the following variables:

PR = Participation Rate;
CR = Cap Rate;
R^* = Effective Cap rate ($PR \neq 1$, $R^*=CR/PR$);
X^A = At-the-money (ATM) Strike price at time i (=100);
X_i^* = Out of the money strike by R^* at time i;
f_i = 1 year continuous forward rate at the beginning of period i;
z_m = m year zero rate;
S_i = Equity Index value at time i.

R^* is the effective cap rate if the PR is not equal to 1. For example, if the CR is 15% and the PR is 90% then the effective cap rate, R^*, is 16.667%. For the investor to achieve a 15% return with a PR of 90% the return on S must actually be 16.667%. Because most investors prefer a $PR \geq 1$ we will assume a $PR = 1$ throughout our analysis. This is commonly done in practice. However, forcing $PR = 1$ often times dictates the CR required to maintain the desired term on the structure, T.

Let P_{i+1} represent the payout or cash flow of a ratchet at time $i+1$. For the call option structure:

Equation 1

$$P_{i+1} = \hat{E}_j[\max[S_{i+1} - X_i^A, 0] - \max[S_{i+1} - X_i^*, 0]]$$

where j represents the time at which the expectations is taken. At issuance of the structure $j=0$. Mathematically, this is the point at which the filtration is define and the expectation is taken with respect to the filtration $I_{j=0}$. Note that the P_{i+1} is simply the payout from a capped call option.

Let N_m represents the compounding effect. This determines the number of CC's controlled by the compound ratchet. Due to the homogeneity of the density function of S we could let the notional amount underlying the CC's change rather than the number of CC's . N_m captures the path dependency while retaining the markovian aspects of S. When the CC's are call options N_m is defined as:

Equation 2

$$N_m = \prod_{i=1}^{m}[1 + PR * \hat{E}_j[\max[\frac{S_i}{X^A_{i-1}} - 1] - \max[\frac{S_i}{X^*_{i-1}} - 1]]] \text{ , } N_0 = 1$$

Let CF_i represent the amount of a cash flow resulting from the CC expiring at time i. Though the CF_i does not actually occur with the compound ratchet it is still required for proper valuation. For call options, CF_i is defined as:

Equation 3

$$CF_i = PR * \hat{E}_j[\max[S_i - X^A_{i-1}, 0] - \max[S_i - X^*_{i-1}, 0]] * N_{i-1}$$

Equations 1-3 are expressed here for the call option structure. However, it is important to realize that they can be expressed as any payout structure that is required as long the CC is European.

When an investor purchases a compound ratchet they pay for an option that has no interim cash flows but ensures that the annual returns on the index are locked in each year *and* rolled into the next annual at-the-money option. For example, if the index were to increase in year one then decrease every year thereafter the investor will still realize a positive return due to that first year but will not receive the cash until expiration. Therefore, the issuer of the ratchet must ensure that these interim cash flows are retained so that at expiration they are available to pay out to the investor. Consequently, the cash flows will earn interest that belongs to the investor and needs to be accounted for – so let S_i be the future value of interest on the interim cash flows. We will refer to this as a "subsidy" for the compound ratchet.[3]

Equation 4

$$S_i = \sum_{i=1}^{T} CF_i * [\exp\{\sum_{k=i}^{T} f_k\} - 1]$$

With these variables defined it is now possible to express the value of the compound ratchet, V_C:

Equation 5

$$V_C = [\sum_{m=1}^{T} N_{m-1} * PR * P_m * e^{-z_m * m}] - S_i e^{-z_T * T} = [PR * \sum_{m=1}^{T} N_{m-1} * P_m * e^{-z_m * m}] - S_i e^{-z_T * T}$$

Equation 5 contains the value of the regular ratchet, V_R, as a special case. Simply let $S_i = 0$ (no subsidy) and $N_{m-1} = 1$ (number of options is always 1 or no compounding). Or,

Equation 6

$$V_R = [PR * \sum_{m=1}^{T} P_m * e^{-z_m * m}]$$

Equation 6 is similar to a series of forward start options (Rubinstein, 1991) except that the amount of the underlying remains constant and the payouts are capped. Both equations 5 and 6 are greatly simplified if we assume a flat term structure for both interest rates and volatilities.

Equations 5 and 6 are evaluated by using the results of Harrison and Kreps (1979) , Cox, Ingersoll, and Ross (1985) and Cox, Ross, and Rubinstein (1979). They showed that if we assume that S follows a Geometric Brownian motion, there is no arbitrage, and *S* is a tradable asset then we can transform the problem to a risk neutral environment. Following the transformation equations 5 and 6 are easily computed for any European type CC. We can also incorporate both the term structures of interest rates and volatility to more accurately evaluate each CC making up the two ratchet structures.

In addition to the valuation formulas all of the "Greeks" of these structures are now easily calculated. These are vitally important for two reasons. First, if the insurance company desires to synthetically replicate the security they must have these sensitivities in order to correctly package the appropriate exchange traded derivatives. Second, in order to implement an effective risk management strategy after the structure has been created these values must be continuously monitored. The issue of risk management is particularly difficult when numerous regulatory requirements are placed on the insurance industry. Regulatory requirements are often evaluated as additional embedded options – and greatly complicate any analysis. Because regulatory requirements are heterogeneous across international borders they are not explained any further here. The sensitivities, however, are analyzed in detail in the next section.

These ratchet structures are normally offered along with a guarantee of the principal (notional) investment or a guarantee of a minimum return on the principal. This characteristic is easily accomplished with the purchase of the appropriate zero coupon bond, $B_Z(T)$. $B_Z(T)$ is most often found using a spread over the current government term structure.[4] For example, if the package were to insure the principal invested (or the principal plus a minimum return), *A*, then the insurance company simply identifies the appropriate zero and takes the remaining monies to purchase the ratchet. Normally, interest rates dictate the participation rate on the ratchet since they are used to price $B_Z(T)$, the amount remaining after fees is then used to purchase the ratchet, $AV = A - B_Z(T) - x$, where *x* represents fees.[5] So if *AV* is not sufficient to cover $V_{C(R)}$ with a $PR=1$, then the bond price dictates the *PR*. However, we can work the other way and solve the problem so that we always have a $PR=1$ by determining what maturity, *T*, needs to be to obtain the desired *PR*. In most interest rate environments, as *T* increases the $B_Z(T)$ will decrease and *AV* will increase thus allowing the insurance company to be able to offer a $PR=1$. T is rarely an integer, consequently for the ratchet structure the insurance company will round up to the nearest year and $PR>1$ – the difference going to the issuer. Another way to obtain the desired *T* and *PR* is by changing the cap rate. As *CR* decreases so does $V_{C(R)}$ allowing the issuer to obtain the desired marketing values and reduce the sensitivities of the ratchet. These tradeoffs will be explained more fully in the next section. In either scenario the primary determinant in the combined structure is the bond and the ratchet piece more or less falls out of the marketing decision. Denote the combined portfolio as Π then the structure to be analyzed by the insurance company is expressed as:

Equation 7

$$\Pi = B_Z(T) + V_{R(C)}$$

Or,

Equation 8

$$\Pi_C = \frac{A}{(1+z_T)^T} + [PR * \sum_{m=1}^{T} N_{m-1} * P_m * e^{-z_m * m}] - S_i e^{-z_T * T}$$

where Π_C is for the compound ratchet . The regular ratchet portfolio, Π_R , is therefore,

Equation 9

$$\Pi_R = \frac{A}{(1+z_T)^T} + [PR * \sum_{m=1}^{T} [P_m * e^{-z_m * m}]$$

The next section addresses the analysis on these portfolios.

ANALYSIS

This section evaluates equations 5 (8) and 6 (9) under varying scenarios. The underlying index is the Standard & Poor's 500 equity index (S), the dividend yield (q) is 1.82%, a Cap Rate (CR) of 15%, a participation rate (PR) of 1, and the term to expiration is five years (T=5). The remaining terms can be found in Table 1, all terms are in percent. The forward rates and the annual volatilities are calculated on a continuous basis. Table 1 is a close approximation to the environment in the United States in the Spring of 1997. We analyze the value of both the ratchets and the portfolio as we alter these underlying values. Specifically, we analyze the effects on value and sensitivities from shifts in the interest rate term structure, the volatility term structure, and the cap rates. Of course, we could easily isolate a particular interest rate as in Ho (1992) or volatility as in Derman and Kani (1994) or Derman, Kani, and Zou (1996) and evaluate the sensitivity of the structure to that variable. However, the purpose here is to establish a more general framework. All figures isolate the effects of changing a single variable while holding all others at their base case values.

TABLE 1
Base Scenario for a Five Year Ratchet Structure

Variable	Symbol	Year 1	Year 2	Year 3	Year 4	Year 5
ZeroRate (%)	z_k	6.11	6.513	6.636	6.73	6.781
Forward (%)	f_k	5.9306	6.688	6.559	6.777	6.752
Volatility (%)	σ_k	17	18	19	20	21

Under this base scenario the regular ratchet has a value of $25.62 and the compound ratchet has a value of $25.26. The metric of value (in this instance U.S. dollars) is not important since the underlying is assumed to be $100, therefore these values actually represent a percentage of original principal with a participation rate equal to 1. For example, the regular (compound) ratchet costs 25.62% (25.26%) of any amount invested in this product. Consequently, we express all subsequent values as a percentage of notional invested. Note that the regular ratchet is more expensive – this is due to the reinvestment of interim cash flows within the compound ratchet structure. It is for this reason that we refer to equation 4 as a subsidy – it actually cheapens the compound ratchet. However, we will show that for CR's over 20% the compound ratchet becomes more expensive than the regular ratchet.

Tables 2a and 2b represent the values of the regular and compound ratchet structures respectively as we shift both the interest rates and volatility from the base scenario. Moving vertically in the tables we are shifting the volatility curve by the amount in the first column and holding all other variables constant. This is equivalent to quantifying the partial derivative of equations 5 and 6 (or 8 and 9) with respect to volatility, otherwise known as Vega or Kappa. Similarly, as we move horizontally we are shifting the interest term structure. This has an identical interpretation, except it is known as rho. The tables show changing values over a 300 bp shift in rates and a 6% change in volatilities. Under the base scenario (the center of the tables) we achieve a $PR=1$ if the fees are less than 3.18% and 3.54% for the regular and compound ratchet, respectively. Fees larger than this will result in a total cost greater than the principal invested. Therefore, larger fees would result in either a lower *PR* or *CR*.

It is interesting to note that as we move from left to right in the tables the ratchet values increase but the portfolio values decrease. This illustrates a very important point: the duration of the bond piece far exceeds the rho of the option piece. Consequently, under a rising rate scenario the option piece partially hedges the portfolio but we would still need to hedge against a fall in portfolio value. Since rho can be easily converted into a duration measure we can establish an aggregate portfolio duration for hedging purposes.

Figures 1A and 1B illustrate the interest rate trade off graphically. This visual offers a better framework to see how rho is positive but the aggregate portfolio duration is negative. Also note the slight curvature on the compound ratchet value as interest rates change that is not present in the regular ratchet. This is due to the compounding effect and the reinvestment of interim cash flows. Note also that as interest rates rise so does the subsidy. If interest rates rise enough the compound ratchet will eventually become cheaper than the regular ratchet.

Figures 2A and 2B are similar to figures 1A and 1B except that we replace shift in interest rates with shifts in volatility. The effect of volatility on the two ratchets is almost identical. This is not surprising since the major difference between the two is how we deal with the interim cash flows which is not effected by the volatility. Additionally, since both are capped the sensitivity with respect to volatility will largely be offset by the “capping” contingent claim. We will show later how the cap rate effects Vega directly. Also note that the portfolio value is perfectly parallel to the ratchet – this is because the fixed income piece is not a function of volatility and consequently is not effected when it changes.

Figures 3A and 3B show the effects of the Cap Rate (*CR*) on the values. Again since the fixed income piece is in no way related to the *CR* it’s value remains unchanged as *CR* changes. Hence the portfolio value relationship. However, notice the extreme effects the cap rate has on the ratchet values. As *CR* decreases the cap strike price approaches the at-the-money strike which greatly reduces the cost. Conversely, as we increase the *CR*, the value increases quickly. For a given *PR*, the *CR* and the term to expiration are the only exogenous variables that the issuer can alter in an attempt to satisfy both the investor and their own risk allowances. Usually, the *CR* is the variable used to obtain a *PR* of 1 for a given term, *T*.

Figures 3A and 3B illustrate another very interesting property between the two structures – the compound ratchet increases in value relative to the regular ratchet as the *CR* increases. As the *CR* increases beyond 20% the subsidy is increasing due to the increased unpaid interim cash flows. However, at the same time the cost reducing effect of the capping option is decreasing more quickly thus causing the compound ratchet to increase is value relative to the regular ratchet. Since cap rates beyond 20% are very uncommon this result is not financially meaningful.

Figures 4A and 4B illustrate how Vega of the ratchets increases exponentially with *CR*. It is interesting to note that the capping contingent claim has a larger Vega for small *CR*’s than the at-the-money contingent claim. This is purely a result of the density function of *S* resulting from the Geometric Brownian motion assumption. Figures 5 and 6 show a similar relationship for the Delta and Gamma of the ratchets, respectively. Note the similarity between the Vega and the Gamma – this is at it should be since both have similar interpretations and must be the same sign. The Delta of the compound ratchet gets increasingly larger than the regular ratchet as the *CR* becomes larger. This is a function of the compounding effect – the number of at-the-money options increases while the effects of the capping options decrease.

As the capping strike moves further out of the money its offsetting effect to sensitivities decreases. This relationship holds for all the sensitivity measures. In other words, the cap hedges the at-the-money contingent claim from extreme moves in any of the underlying variables. In the extreme as the *CR* approaches zero the capping option and the at-the-money option are the same and completely offset each other. This result is shown in figures 3 through 6. The risk to the issuer decreases as the *CR* decreases. However, so does the potential payoff to the investor. In order for the risk/reward tradeoff to be met the ratchet value must decrease accordingly. Indeed it does – thus obeying perhaps the oldest principal of financial economics.

CONCLUSION

We developed a generic pricing methodology to price the ratchet structure. Though the model was developed where the payoff profiles were call options, the same development can be used for any payoff as long as the CC's are of the European type. The results of the model are intuitive and have proven accurate when compared to industry standards.

We showed that as the *CR* increases (decreases) both the price and sensitivities of the ratchet increases (decreases). This result has important risk management and synthetic replication consequences. Additionally, it was shown that the primary determinant behind the structure is the fixed income piece of the total structure; this piece dictates the PR or the CR depending on which factor is more important to the issuer. The fixed income portion also dominates the ratchet portion as interest rates change. Consequently, the issuer must be aware of this property in a rising interest rate environment when early withdrawals may be of concern.

The model introduced here is easily adapted to include many of the regulatory requirements placed on insurance companies. These additional constraints are included as embedded options in the structure. As long as we can use efficient exercise criteria they are easily incorporated. Future research will include extending this model to include both path dependencies and American style contingent claims. American style ratchets could be evaluated using a finite difference approach with resetting boundary conditions. Path dependent ratchets could be evaluated using a Monte Carlo approach and having the resetting aspect captured in each simulation vector.

ENDNOTES

1. The resetting or ratcheting period does not have to be a year. However, we assume an annual resetting period because that is the period most commonly used throughout the insurance industry.

2. Even if this were not the case, we could artificially impose it by simply altering the number of options being transacted. This is due to the homogeneity of degree one of the CC model with respect to the underlying and the strike.

3. An alternative way to look at this property is to imagine that the issuer actually distributed the cash at the end of each year. Then the investor could invest the proceeds accordingly. However, the rate of interest on these cash flows must be the risk free rate in our risk neutral setting.

4. The range of spreads frequently used are from 15 to 50 bps. Throughout this analysis we will use a spread of 25 bps.

5. A fee structure must also be accounted for in the pricing scheme. Throughout the analysis we use a fee range of 3-4%.

FIGURE 1A
Regular Ratchet and Corresponding Portfolio Value as Interest Rates Shift from Base Scenario

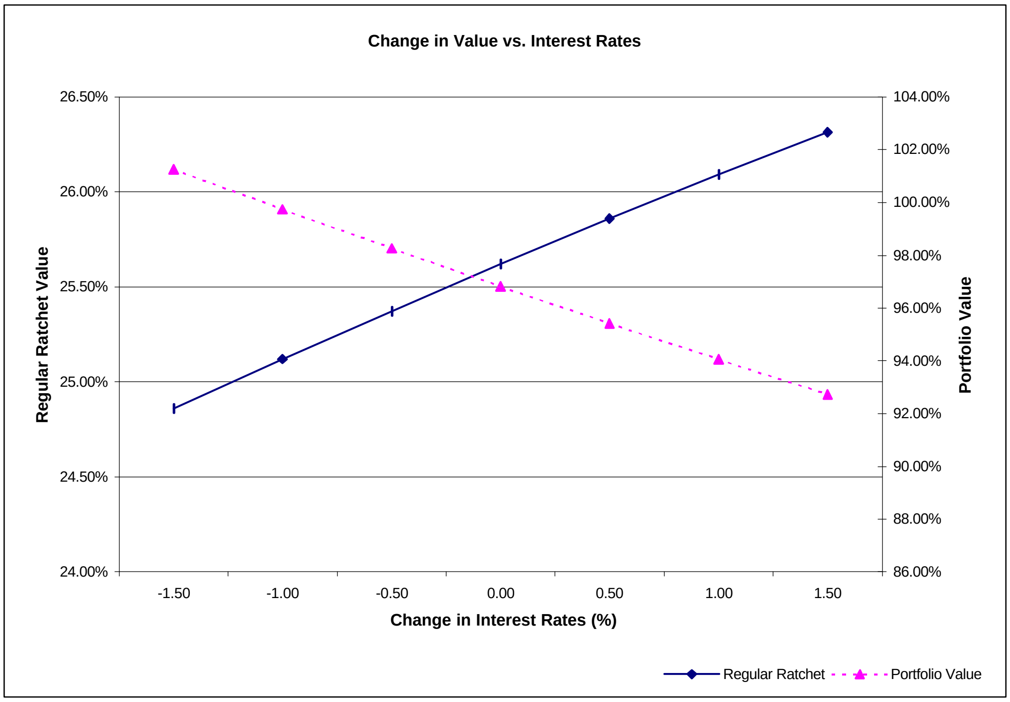


FIGURE 1B
Compound Ratchet and Corresponding Portfolio Value as Interest Rates Shift from Base Scenario

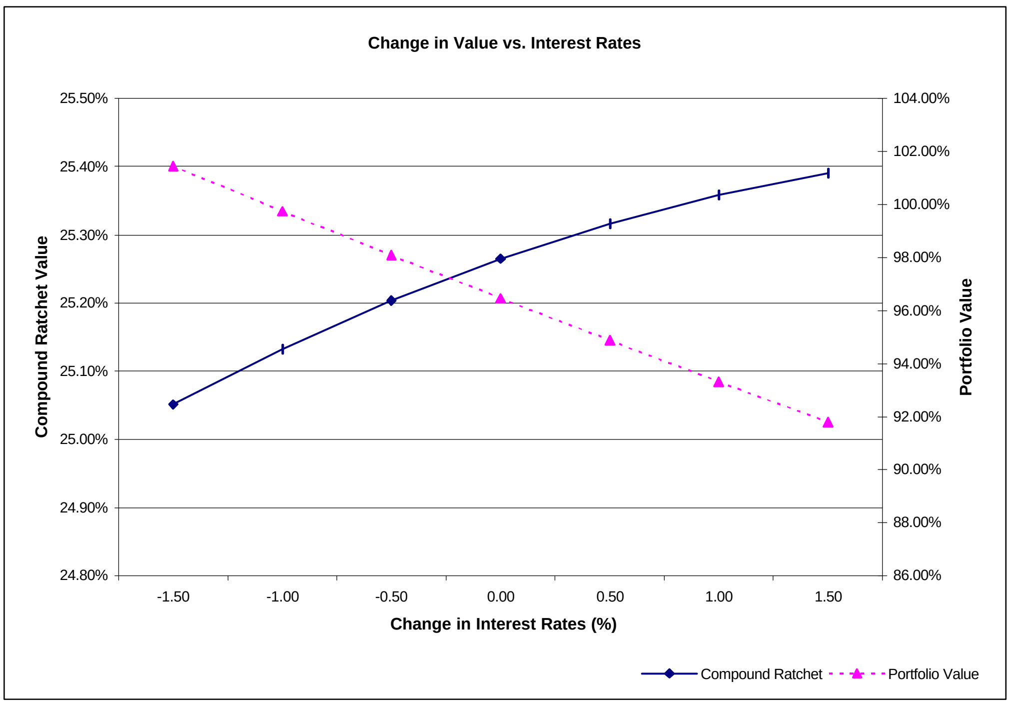


FIGURE 2A
Regular Ratchet and Corresponding Portfolio Value as Volatility Shifts from Base Scenario

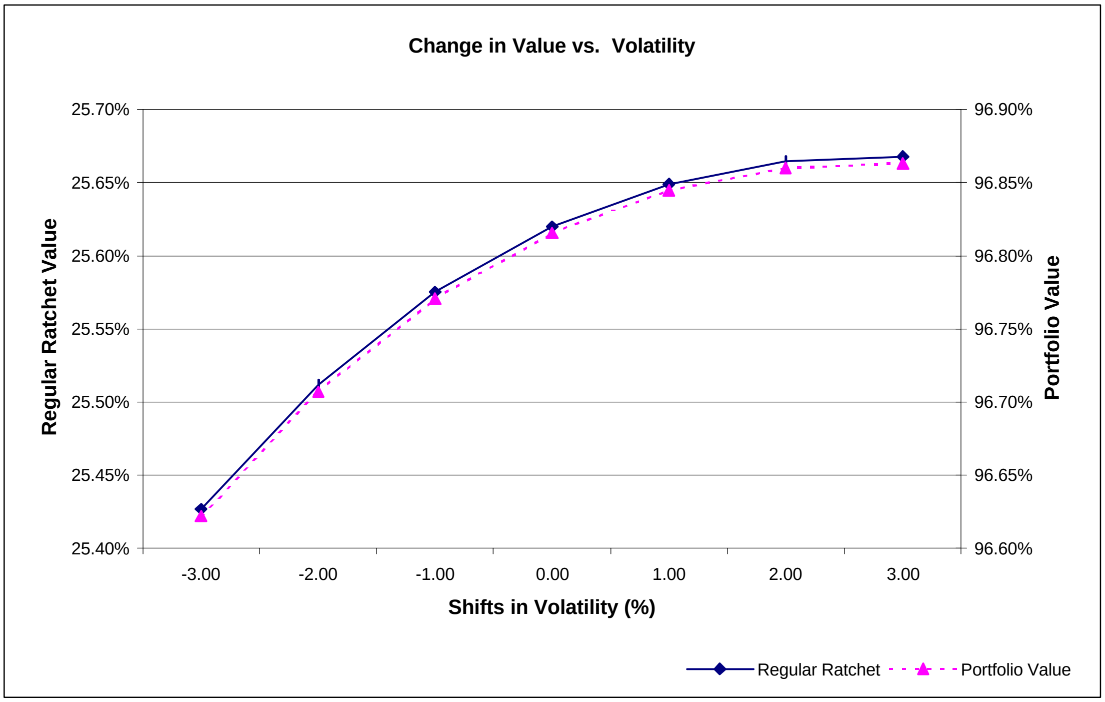


FIGURE 2B
Compound Ratchet and Corresponding Portfolio Value as Volatility Shifts from Base Scenario

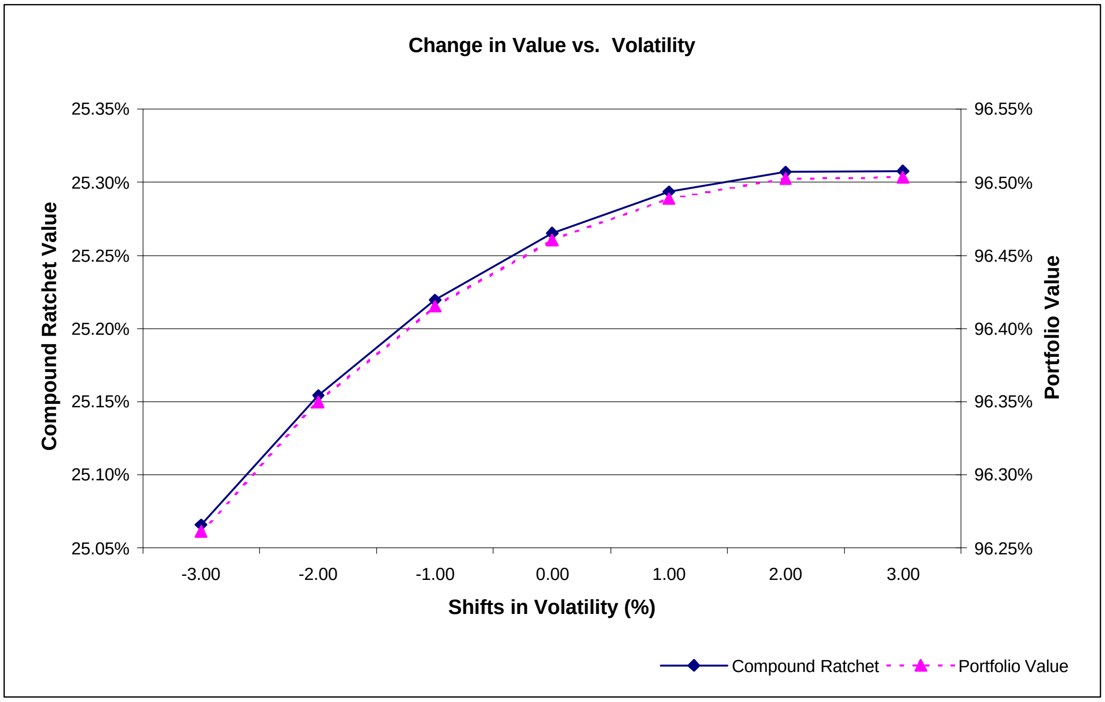


FIGURE 3A
Regular Ratchet and Corresponding Portfolio Value as Cap Rate Shifts from Base Scenario

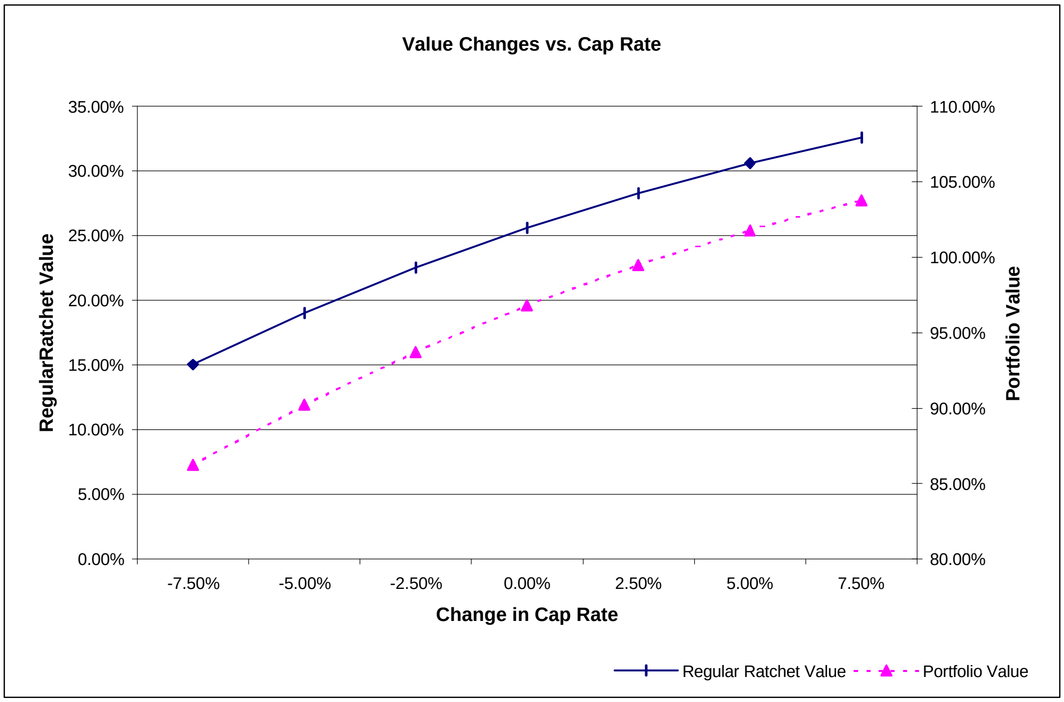


FIGURE 3B
Compound Ratchet and Corresponding Portfolio Value as Cap Rate Shifts from Base Scenario

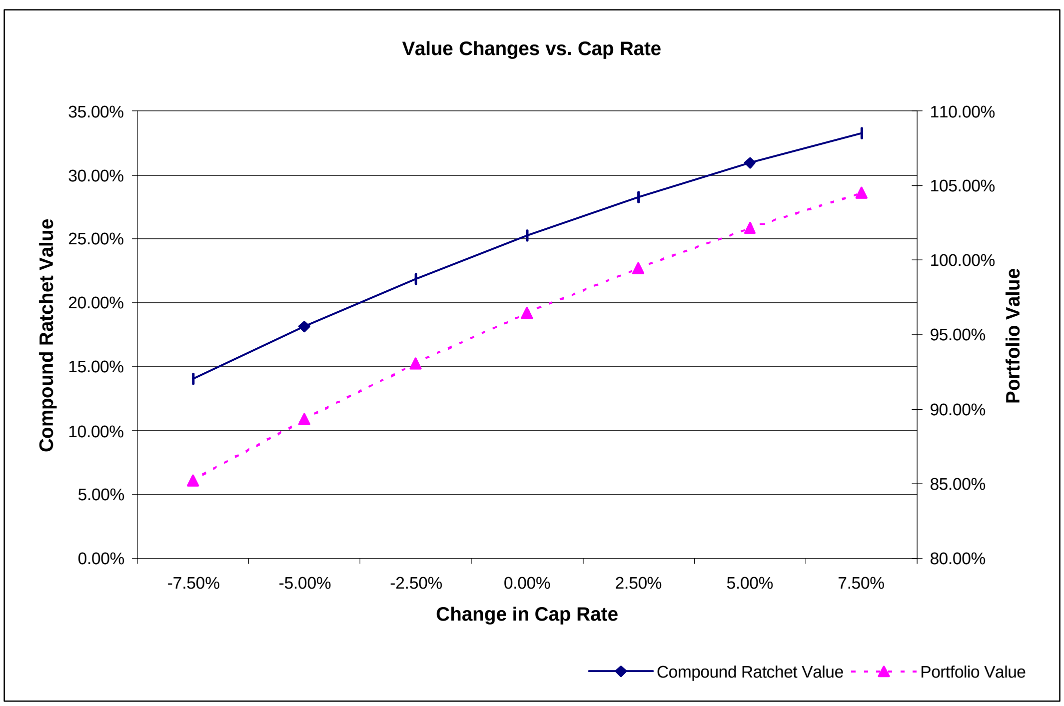


FIGURE 4A
Vega of Regular Ratchet as Cap Rate Shifts from Base Scenario

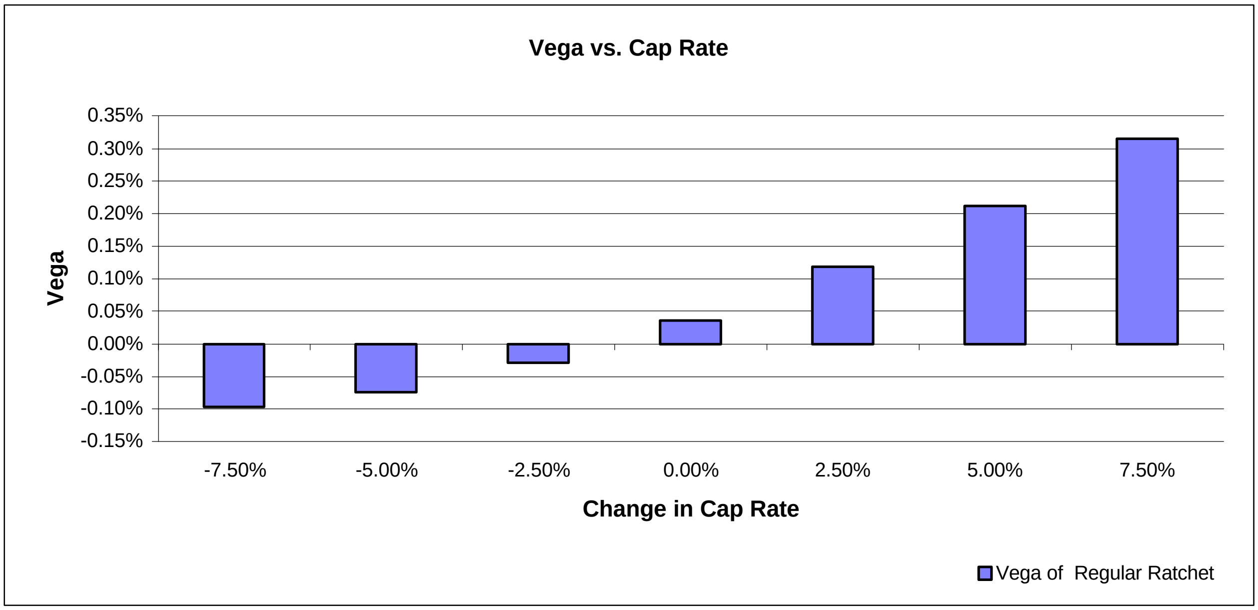


FIGURE 4B
Vega of Compound Ratchet as Cap Rate Shifts from Base Scenario

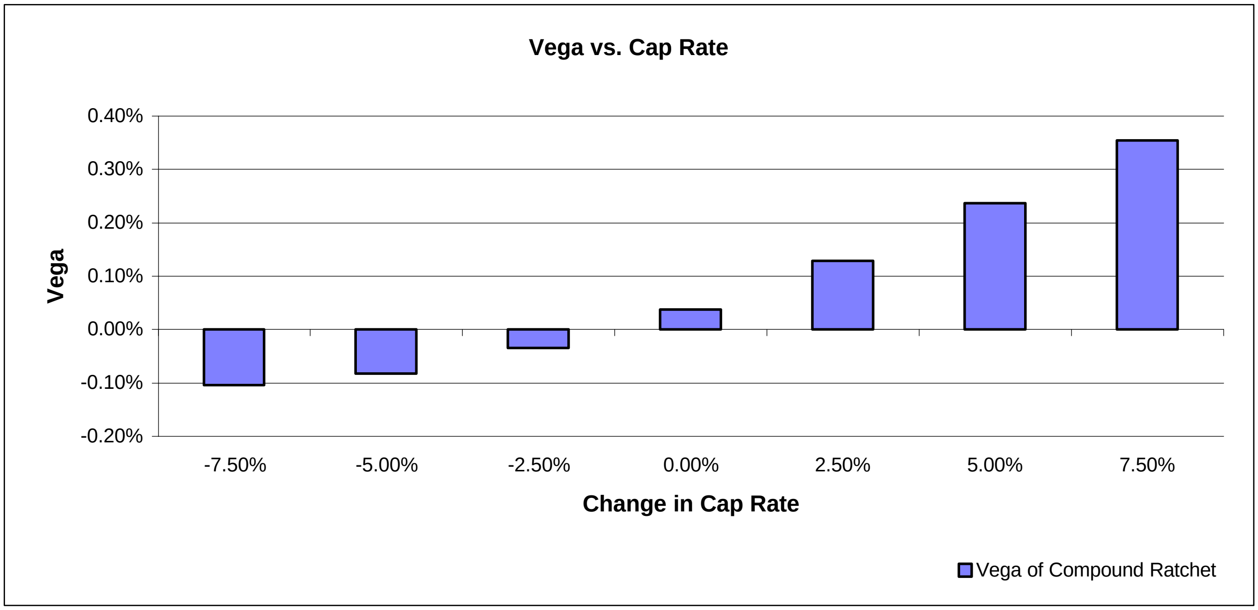


FIGURE 5A
Delta of Regular Ratchet as Cap Rate Shifts from Base Scenario

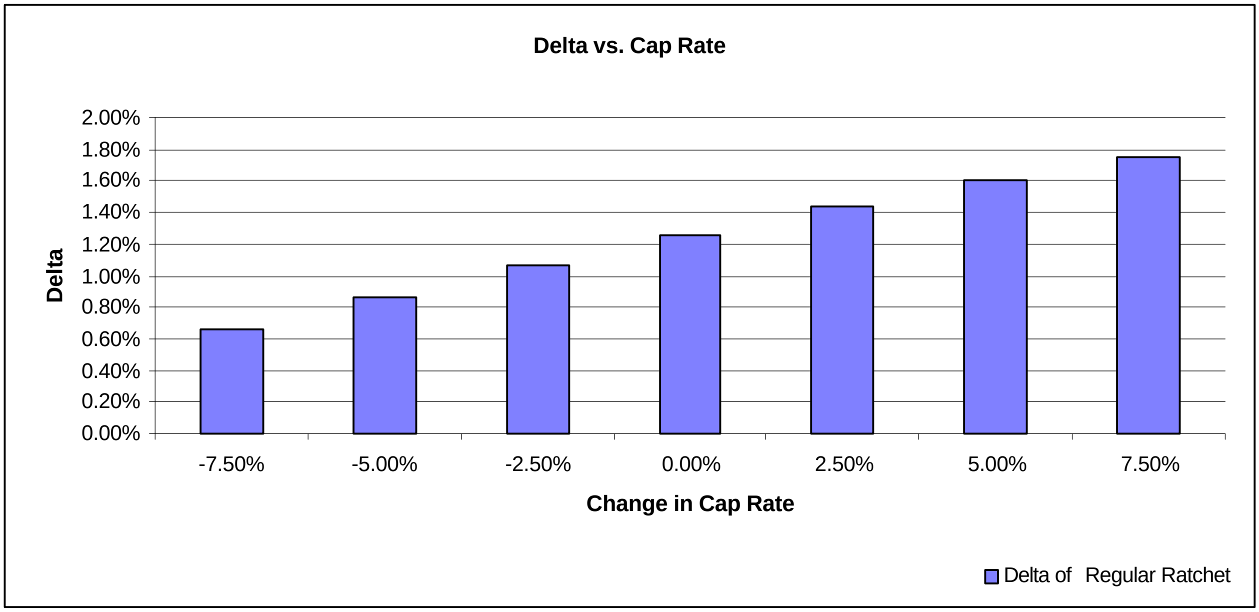


FIGURE 5B
Delta of Compound Ratchet as Cap Rate Shifts from Base Scenario

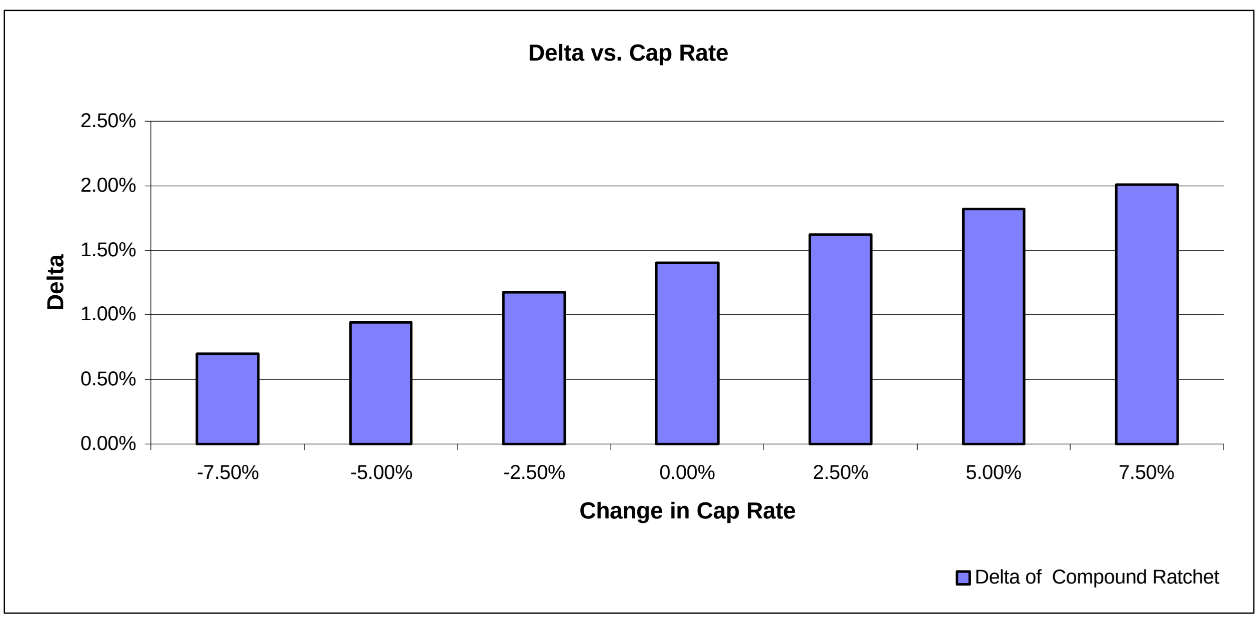


FIGURE 6A
Gamma of Regular Ratchet as Cap Rate Shifts from Base Scenario

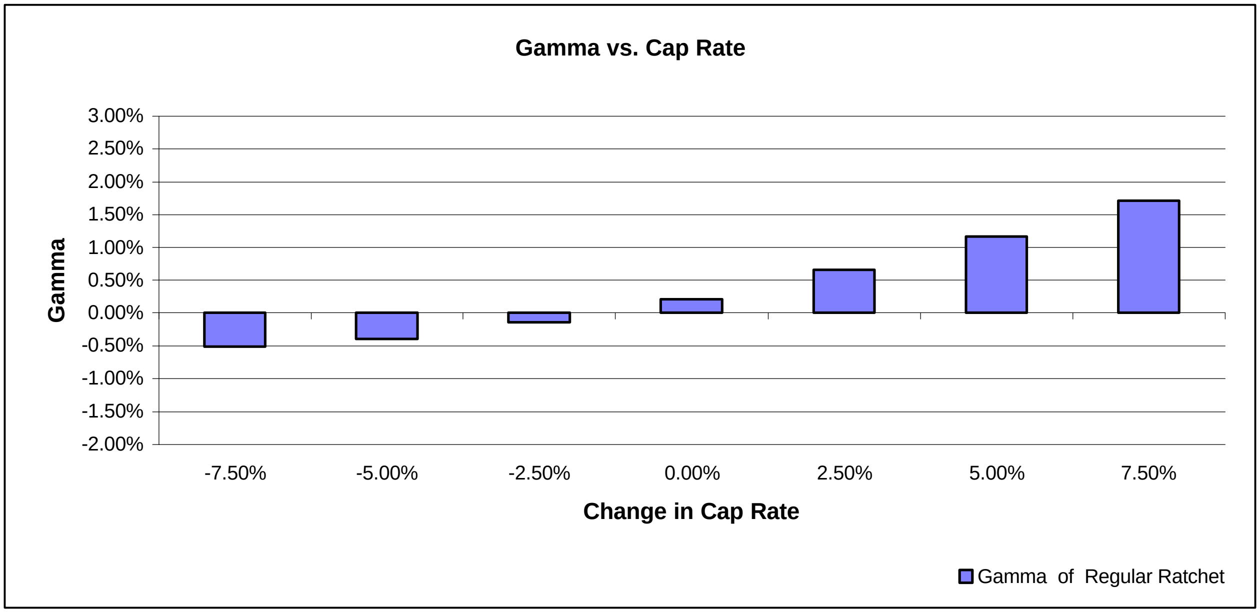
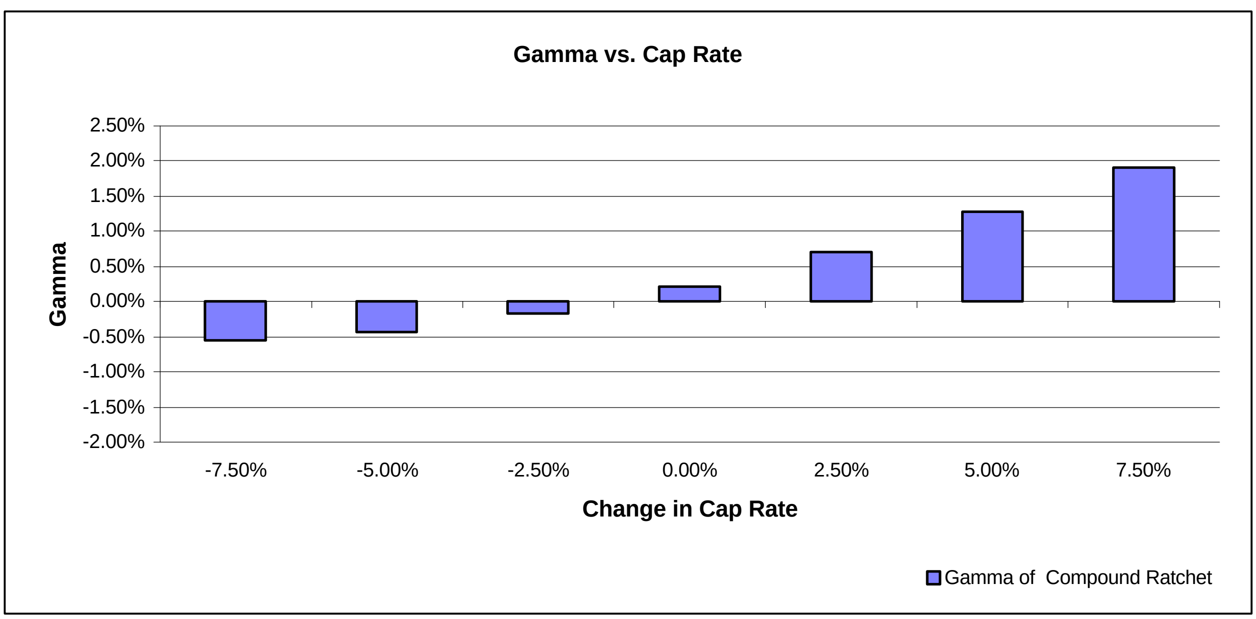


FIGURE 6B
Gamma of Compound Ratchet as Cap Rate Shifts from Base Scenario



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