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EXECUTIVE STOCK OPTIONS: RISK AND INCENTIVES

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Abstract

We perform comparative analysis on an optimal compensation contract comprised of stock options and a fixed salary for two types of managers. In a principal-agent framework, we obtain the pareto optimal solution for the level of stock options in a manager's compensation contract. We then analyze the sensitivity of this optimal contract to changes in the firm's performance, the volatility of the firm's payoffs, and the manager's risk attitudes. In an uncertain environment, we find that shareholders can reduce the volatility of their ownership claim by including stock options in a manager's contract. We also find that a compensation contract with options remains near pareto optimality in the face of changing business conditions. Shareholders are spared the costs of continuously re-contracting with managers to keep their incentives properly aligned with shareholders.

INTRODUCTION

Ever since the paper of Jensen and Meckling [11], attention has been increasingly given to agency theoretic implications to modern financial theories. Similar issues are also addressed in principal-agent problems spun by the important works of Shavell [17], Harris and Raviv [9], and Holmstrom [10] which recognize the asymmetry regarding what the principal and the agent bring into or invest in the firm. Both veins of research are interested in increasing the firm owner's welfare thru incentive alignment. In the former, one would be interested in incentive alignment by reducing agency costs, and in the latter, by designing an optimal incentive contract. Managerial compensation in the form of fractional ownership in the firm is an appealing and a feasible alternative for either school. The relevance of using ownership is underscored by the fact that the majority of existing executive compensation packages have a significant proportion of their remuneration in the form of stocks and stock options. Unlike stocks, stock options allow fractional ownership in the firm only in the event that some favorable future states occur. Although it is still a matter of debate, some empirical works such as those of Aggrawal and Mandelker [1] and Morck, Shleifer and Vishny [15], have found that managerial security holdings on the firm have reduced agency costs and increased firm value. These empirical results together with the results of McConnell and Servaes [14] suggest that managerial firm ownership and firm value do not follow a monotonic relationship. Their results suggest that there is some optimal level of fractional ownership in the firm, after which the principal or shareholder does not benefit from compensating the manager with more ownership claims to the firm.

One of the objectives of this paper is to obtain a closed-form solution for the optimal level of stock options provided to the manager in a principal-agent framework. It goes further and does a comparative analysis on the sensitivity of the optimal contract to small perturbation to the value of parameters in the model, such as the firm's expected pay-offs, the volatility of the firm's pay-offs, and the manager's risk attitutdes. The sensitivity of the contract are differentiated and compared for two types of managers who are distinguished from each other depending on whether or not stock options elicit managerial effort to improve firm performance. Stock options will subsequently affect the distribution of ownership claims and inferences are drawn regarding the benefits to shareholder ownership.

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This paper suggests that while the manager, whose effort is motivated by stock options, requires a higher level of stock options, hiring these managers to run a firm in a dynamic environment promotes more stability in the firm's ownership structure. Similarly, while the manager whose effort is invariant to stock options requires lesser amount of stock options, the ownership structure of firms run by these manager will be more unstable. That is, the level of original shareholder's ownership in a firm operating in a dynamic environment has more uncertainty.

Realizing the more dynamic environment under which firms operate, the optimal level of stock options granted at the signing of the contract could easily be far from the optimum prior to its maturity five to ten years later, thereby defeating the initial incentive objectives of the contract at the time it was signed. As such, undesirable incentive effects may result. To maintain the proper incentives, contracts need to be periodically adjusted. It comes as no surprise that we frequently observe Board of Directors making adjustments to the long-term compensation contracts of their executives. Among those compensation items that are frequently adjusted are the stock options. The later part of the paper provides comparative analysis on the sensitivity of a compensation contract with stock options. It compares the necessary level of adjustment made in response to different changes in the operating parameters of the contract so that pareto optimality is maintained. The analysis in this paper suggests that the manager whose effort is motivated by contingent ownership claim (i.e., the stock options) requires lesser adjustments as a result of parametric changes. For the same change in expected firm cash flow, the manager whose effort is motivated by stock options will require less adjustment to his original level of stock options. It is also conceivable that the manager's attitude can change over the term of the contract. For the same increase in risk aversion by both kinds of managers, the manager motivated by stock options will require less of an increase in the level of stock options. Moreover, and rather interesting, when the volatility of firm earnings increases due to some externality, the manager whose effort is motivated by stock options requires a decrease in his stock options but the manager whose effort is invariant to stock options requires an increase in his stock options. Given that the contract periods are often over extended periods of time and changing parameters are likely the norm rather than the exception, the sensitivity analysis suggests that over an extended period of time, hiring the manager whose effort are motivated by the level of stock options, though initially requiring a higher level of stock options, benefits the shareholders by reducing the volatility of his share ownership in the firm. In some instances, the shareholder ends up giving less of his ownership in the firm than was initially anticipated.

That the levels of adjustments are less, or that it takes fewer and less amount of adjustments to maintain pareto optimality for contracts drawn with managers with incentive effects, also suggests that such contracts are more likely to remain close to pareto optimality over a longer period of time than a contract drawn with a manager with no incentive effects. This suggests that there will be a difference in the frequency of recontracting for these two types of managers. Recontracting and adjustments in the executive compensation contract will be more frequent for executives whose effort is indifferent to stock options.

Section I sets up the model. Managers are assumed to have positive marginal utility for income but have a dislike for effort. Stock options are provided as an incentive for the manager to perform at or above a certain level of performance as measured by the firm's cash flow which has a random component and a component resulting from managerial effort. A common performance measure is firm value. When the desired firm value is achieved or exceeded, the manager gets rewarded with a fraction of the firm ownership. Generally speaking, in the event of favorable firm performance, a manager with stock options will experience a jump in his income when he becomes a fractional owner of the firm and is enabled to particpate in whatever the firm's upside gains end up to be by the end of the contract period. Certainly, there are managers whose effort to increase firm performance can be motivated by the possibility of firm ownership or large upside gain, particularly if they believe their effort can improve the chance of favorable firm performance or event to occur. This kind of manager is identified as a Type I manager. On the other hand, there can be managers whose effort dedicated to increase performance are hardly influenced by stock options. Such indifference might be present with managers or executives about to retire or, for other reasons, among managers who just do not find value in the incremental earnings from compensation in the form of ownership. This latter case can exist if regulations and/or the negative signalling effect of selling of his shares preclude the manager from the economic benefit of firm ownership gained through the exercise of his stock option at the end of the contract period. This managers is identified as a Type II manager.

The analysis in this paper, the pareto optimal contracts and the comparative analysis, focuses on these two types of managers. These managers are restricted from hedging and trading their options during the contract period. The obtained closed-form solutions derived in Section II consist of the stock option and fixed salary comprising the optimal contract so that the manager spends effort to maximize shareholder's welfare. The solution depends on parameters, mentioned earlier, such as the expected firm cash flow and volatility which are easily measured. Other parameters of the contract, such as the shareholder's and manager's risk attitude, are endongeous to the model and are not directly quantifiable but for which their direction of change are generally discernable. As such, our result is able to further provide useful insight towards a better understanding of compensation contracts using executive stock options. Beck and Zorn [2] investigated the efficient allocation of stocks between the principal and the agent. This paper is distinguished from theirs by studying options in which, unlike stocks which award firm ownership from the time the contract is signed, fractional firm ownership is contingent upon performance. As such, fractional ownership in the firm is not a certainty with stock options.

THE MODEL

To simplify our analysis, we allow one shareholder to represent all the shareholders of a firm with uncertain cash flows. At time 0, the representative shareholder and a manager design an executive compensation package that will maximize the expected utility of the shareholders while maintaining the reservation price of the manager. The compensation package is made up of a fixed salary and an option to purchase a fractional share of the firm at some future time, t=1, only if the firm's net cash flow exceeds a certain level. Thus, a riskless and a risky income sources comprise the managerial income portfolio and are endogeneous in our model. Both the shareholder and the manager have positive marginal utility for wealth, and both have concave utility functions. Thus, risk bearing will play a major role in the design of the compensation package and the consequent allocation of time 1 ownership in the firm via the stock option. Aside from being risk averse, the manager has an increasing dislike for effort.

The manager is restricted from trading his stock options prior to its maturity which occurs at t=1. This trading restriction precludes the manager from engaging in riskless hedging strategies using his executive stock options. Although most executive stock options have staggered maturity dates, lumping the maturity dates of all executive stock options to a single future date does not affect the general outcome of our results.

At time 0, the representative shareholder of the firm and the manager negotiate a contract which relates the manager's time 1 compensation to the firm's time 1 cash flows. The firm's time 1 cash flows have a random component and a component that is directly the outcome of managerial effort. For simplicity we denote the firm's time 1 cashflow as the sum of these components, $\theta + e$. θ is the random variable. It is the component of the firm's cashflow which is beyond the control of the manager. The level of managerial effort, $e = e(w,c)$, is a function of the compensation package. This compensation package comprises a state-independent salary, w, plus call options on a fraction, c, of the total shares of the firm where $0 \leq c \leq 1$. The shareholder can not give more than he owns. As such, c cannot exceed 1 nor be less than zero. The share constitute a claim on the firm's time 1 cash flows, $\theta + e$, net of the manager's state-independent salary, so its total price is $\theta + e\text{-}w$. Let X be the time 1 exercise price of the option on the firm. When the time 1 net firm cash flow is equal to or exceeds X, the manager can exercise his stock option and claim a share c of the firm upon payment of the exercise price of cX to the shareholder. Thus when $c(\theta + e\text{-}w)\text{-}cX > 0$, the manager gains from exercising his option.

Let θ^0 be a borderline state. Let l denote the set of low states where the firm's net cash flow, $\theta + e - w < \theta^0$ so that he does not exercise his options and therefore receives an income $i_l(\theta) = w$. Let h denote the set of high states when net firm cash flow, $\theta + e - w \geq \theta^0$, so that he exercises his option and receives $i_h(\theta) = w + c(\theta + e\text{-}w) - c X = w + c(\theta + e\text{-}w - X)$.

In other words, the manager's total income, contingent on the state θ and the level of e, is given as follows:

(l) If $\theta + e\text{-}w < \theta^0$, then $i_l(\theta) = w$
(h) If $\theta + e\text{-}w \geq \theta^0$, then $i_h(\theta) = w + c(\theta + e\text{-}w - X) = (1 - c)w + c(\theta + e - X)$

At the borderline state, $\theta^0 = X\text{-}e+w$, exercising the option gives the manager $i_h(\theta^0) = i_l(\theta^0) = w$.

The Manager's utility function is composed of his expected utility for income in the low states, $\pi_l u(w,e)$, and his expected utility in the high states, $E_h[u((1\text{-}c)w + c(\theta+e\text{-}X),e)]$. π_l is the probability of the low state and $E_h[.]$ is our expectation operator for the high state.

For any given w,c, the manager's optimal choice of effort level is the solution to his expected utility maximization problem given by:

Equation 1

$$\underset{e}{MAX}\ R(w,c) = \pi_l u(w,e) + E_h[u((1\text{-}c)w + c(\theta+e\text{-}X),e)]$$

where $E_l[u_e]+E_h\,[u_e]+E_h\,[u_i c]=0$ and $E_l[u_{ee}]+E_h[u_{ee}]+E_h[u_{ii}c^2]<0$ are the first and second order conditions, respectively.

Applying the Implicit Function Theorem at the optimal level of *e*, it follows that:

Equation 2

$$\frac{de}{dc}=-\frac{F_c}{F_e}=-\frac{-E_h\,[u_{ii}c\bar{z}_h\,]+E_h\,[u_i\,]}{E_l\,[u_{ee}\,]+E_h\,[u_{ee}\,]+E_h\,[u_{ii}c^2\,]}>0$$

Similarly,

Equation 3

$$\frac{de}{dw}=-\frac{F_w}{F_e}=-\frac{E_h\,[u_{ii}c(1-c)]}{E_l\,[u_{ee}\,]+E_h\,[u_{ee}\,]+E_h\,[u_{ii}c^2\,]}<0$$

Since *w* and *c* are substitute sources of income, an increase in *c* will be accompanied by a decrease in *w*. Hence the net change in managerial effort from an increase in *c* will depend on the incentive effects from the increase in stock options and from the decrease in the fixed salary that was substituted for the additional stock options. Furthermore, it will depend on the manager's marginal rate of substitution of the fixed salary received in all states for the contingent form of compensation. Theorem 1 proves that the manager's compensated effort supply elasticity to an increase in stock options is positive. This suggests that the net effect of an increase in the level of stock options, including the conditions stated above, is to increase the level of managerial effort. This confirms the positive net effect of stock options on the level of managerial effort.

Theorem 1: Given a risk averse manager with an increasing dislike for effort, his compensated effort supply elasticity to increases in stock options is positive and is given by:

$$\varepsilon=\frac{c}{e}\left(e_c-\frac{E_h[(\theta+e-w-X)u_i\,]}{E_l\,[u_i\,]+E_h[(1-c)u_i\,]}e_w\right)>0$$

Proof: Consider the effect of a small change *w* and *c* on the manager's expected utility. A small change in the fixed salary, δw, on the manager's expected utility is given by:

Equation 4

$$\frac{\partial R(w,c)}{\partial w}\delta w=\{E_l[u_i]+E_h[(1-c)u_i]\}\delta w>0$$

Similarly[2], the effect of a small change in the level of stock option, δc, is:

Equation 5

$$\frac{\partial R(w,c)}{\partial c}\delta c=\{E_h\,[(\theta+e-w-X)u_i]\}\delta c>0$$

Substituting both equations (4) and (5) into the definition for the slope of a compensated effort supply function, we obtain:

Equation 6

$$\left(\frac{\partial e}{\partial c}-\frac{\partial e}{\partial w}\frac{\frac{\partial R(w,c)}{\partial c}\delta c}{\frac{\partial R(w,c)}{\partial w}\delta w}\right)\equiv\left(e_c-\frac{E_h[(\theta+e-w-X)u_i\,]}{E_l(u_i\,]+E_h[(1-c)u_i\,]}e_w\right)$$

where upon multiplying both sides by *c/e* and applying the results of equations (2) and (3), it follows that the compensated effort supply elasticity, ε, due to an increase in *c* is:

Equation 7

$$\varepsilon = \frac{c}{e}\left(e_c - \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}\ e_w \right) > 0 \qquad \text{Q.E.D}$$

Managers who have positive effort supply elasticity are referred to as Type I managers. These are managers whose level of efforts are motivated by the level of stock options and fixed salary in his compensation package.

It is assumed that the manager will exercise his stock options if the high states occur. As such, the shareholder's income when their firm is managed by a Type I manager is contingent on state θ at time = 1 and the managerial effort. The shareholder's income, I(θ), is as follows:

(l) If $\theta + e\text{-}w < \theta^0$, then $I_l(\theta) = \theta + e\text{-}w$
(h) If $\theta + e\text{-}w > \theta^0$, then $I_h(\theta) = (1\text{-}c)(\theta + e\text{-}w) + cX = (1 - c)(\theta + e) + cX - (1\text{-}c)w$

The shareholder's problem is:

$$\underset{w\ c}{MAX}\ H(w,c) = E_l\,[U(\theta + e\text{-}w)] + E_h\,[U((1 - c)(\theta + e\text{-}w) + cX)]$$

subject to: $R(w,c) = u^*$

where $R(w,c) = \pi_l u(w,e) + E_h[u((1\text{-}c)w + c(\theta + e\text{-}X),e)]$ is the manager's utility function given earlier by equation (2) and u^* is the manager's reservation price. u^* represents the manager's expected utility from the best alternative employment elsewhere. The closed form solution to the shareholder's optimization problem gives the optimal level of stock options, *c*, and fixed salary, *w*, comprising the optimal compensation contract. It is given by Proposition 1.

THE PARETO OPTIMAL CONTRACT

Proposition 1: If compensation has an incentive effect on the level of managerial effort, then to a second order approximation, the Pareto optimal level of stock option *c* satisfies:

Equation 8

$$\frac{\bar{z} - (1-c)AV_h}{B + (1-c)} - \frac{\bar{z} - caV_h}{b + (1-c)} = \frac{e\varepsilon}{c}$$

where V_h is the upper semi-variance of the firm's revenue in state *h*, $\bar{z}$ is the expected incremental firm cash flow in the high state, *a* and *A* are the manager's and shareholder's level of risk aversion, *b* and *B* are the marginal rate of substitution of low and high income state incomes for the manager and the shareholder, respectively; *e* is the level of managerial effort, *c* is the fractional claim of the manager on the firm from his options, and ε is the manager's compensated effort supply elasticity.

Proof: The proof to Proposition 1 is available in Appendix.

Remarks: The optimal level of *c* is exact for managers and shareholders whose utility functions are quadratic. $V_h \equiv E_h[(\theta - \bar{\theta}_h)^2]$ is the upper semi-variance of the firm's revenue in state *h*. It is a measure of the volatility of the firm's earnings; $\bar{z} = \bar{\theta}_h + e\text{-}w\text{-}X$ is the expected incremental firm cash flow in the high state . (Recall that at the borderline state, $X = \theta^0 - e + w$). It is the difference between the firm's value in the high state and the exercise price. $a \equiv -[u_{ii}\ (i_h(\bar{\theta}))]/u_i\ (i_h(\bar{\theta}))$ and $A \equiv -[U_{II}\ (I_h(\bar{\theta}))]/U_I\ (I_h(\bar{\theta}))$ are the manager's and shareholder's Arrow-Pratt measures of absolute risk aversion, respectively; and $B = E_l[U_I]/E_h\,[U_I]$ and $b = \pi_l u_i/E_h[u_i]$ are the marginal rate of substitution between high income state and low income state for the shareholder and manager, respectively. caV_h and $(1\text{-}c)AV_h$ can be interpreted as the marginal risk premia assigned by the manager and shareholder to the firm's returns, respectively. So that $\bar{z} - caV_h$ and $\bar{z} - (1\text{-}c)Av_h$ are like risk-adjusted cash flows of the firm from the point of view of the manager and the shareholder, respectively.

If stock options have no incentive effects so that manager's level of effort is unaffected by the level and combination of stock options and the fixed salary *w* in the compensation package, then the pareto optimal compensation package for the manager of a risky firm is given by Proposition 2. We shall identify these kinds of managers whose effort supply elasticity is equal to zero as Type II managers. These are managers whose level of effort is invariant to the level of stock options and fixed salary in his compensation package. This lack of incentive effect can exist if regulations and/or the negative signalling effect of selling of his shares after the contract period preclude the manager from the economic benefit of firm ownership gained through the exercise of his stock option at the end of the contract period.

Proposition 2: If compensation has no incentive effects, then, to a second order approximation, the Pareto optimal level of *c* satisfies:

Equation 9

$$\frac{\bar{z}-(1-c)AV_h}{B+(1-c)} = \frac{\bar{z}-caV_h}{b+(1-c)}$$

where the firm's incremental cash flow in the good states, $\bar{z} = \bar{\theta}_h - w - X$, and the firm's volatility, V_h, have no component attributed to managerial effort. The parameters *B, b, A* and *a* are as identified earlier and carries a similar interpretation as those noted in Proposition 1.

Proof: The proof to Proposition 1 is available in Appendix.

Remarks: caV_h and $(1-c)AV_h$ are the risk premia for the manager and the shareholder. Hence, $\bar{z} - caV_h$ is the risk-adjusted incremental firm cash flow for state *h* as viewed by the manager. While $\bar{z} - (1-c)AV_h$ is the risk-adjusted incremental firm cash flow for state *h* as viewed by the shareholder. These risk-adjusted firm cash flows suggest that giving ownership in the form of a stock option is a means for risk shifting. The optimality condition, equation (9), clearly shows that as *c* increases, caV_h, the risk premium of the manager increases and that of the shareholder, $(1-c)AV_h$, decreases. Given that fixed salary and stock options are subtitutes, when the manager receives a lower fixed salary in exchange for more stock options, the fixed cost to the shareholder in all states is reduced. This reduced cost increases the shareholder income sheltered from uncertainty in all states. The risky component of the shareholders income in all state decreases, thereby reducing the risk borne per unit of income. On the other hand, the increase in the level of stock option in the managers compensation packages increases the risk per unit income borne by the manager.

COMPARING THE OPTIMAL STOCK OPTION FOR THE TYPE I AND TYPE II MANAGERS

When $\varepsilon \neq 0$, the expression for the closed form solution for c is given by equation (8). It reduces to equation (9) when $\varepsilon = 0$. Using the same parametric values for both, the optimal level of *c* for a Type I manager will differ from a Type II manager. In the discussion below, we show that the optimal level of *c* for a Type I manager when $\varepsilon \neq 0$ will be greater than that for a Type II when $\varepsilon = 0$, ceteris paribus. However when the comparative analysis is done inthe nest section, although the Type I manager needs a higher level of stock options than a type II manager, the Type I manager will require less frequent or lesser amount of compenation adjustments when operating in a dynamic environment where parameters are likely to change.

Let $C=1-c$, where *C* is the fraction of the firm retained by the shareholder[3]. This simplifies equation (9) into a more algebraically manageable function.

Consider the case of a Type II manager whose effort is invariant to the level of stock options, i.e., $\varepsilon = 0$. As a function of *C*, the optimal solution for this manager is embodied in a familiar quadratic function whose solution can be illustrated graphically. It is the equation of a parabola:

Equation 10

$$0 = C^2(a+A) + C(Ba+bA-a) + (B-b)z/V-Ba$$

where $z \equiv \bar{z}, V \equiv V_h$ (For notational convenience, the subscript are dropped). Its vertical intercept is $(B-b)z/V- Ba <$

0. The relevant[4] solution to this quadratic function is the positive horizontal intercept given by C^* in Figure 1. The optimal level c^* can be readily obtained by noting that $1\text{-}C^*=c^*$.

On the other hand, the optimal level of C when stock options motivate positive managerial effort ($\varepsilon \neq 0$), as the case for the Type I manager, is given by the solution to a nonlinear function made up of (i.) a non-linear function which is asymtotic to $C=1$, is a decreasing function of C and has a negative vertical intercept $-e\varepsilon Bb < 0$ and, (ii.) a parabolic function whose vertical intercept is $(B\text{-}b)\, z/V - Ba < 0$. This was obtained by rewriting equation (8) as a function of C. Where upon substitution, (i.) is embodied on the left-hand side and (ii.) is embodied in the right-hand side in the function below

Equation 11

$$-\frac{e\varepsilon}{V}\left\{\frac{Bb}{1-C}+\frac{C(B+b)}{1-C}+\frac{C^2}{1-C}\right\} = C^2\,(a+A) + C(Ba+bA\text{-}a) + (B\text{-}b)\, z/V\text{-}\, Ba$$

The solution to this non-linear function can be graphically illustrated by the intersection of two lines: one line represents a parabola with a vertical intercept of $(B\text{-}b)\, z/V\text{-}\, Ba < 0$; and the other is a line asymtotic to C=1, is a decreasing function of C, *and* has a negative vertical intercept $-e\varepsilon Bb < 0$. When $\varepsilon \neq 0$, the solution is the intersection of these two lines and is identified in Figure 1 by C^{e*}.

Using the same parametric values, for the quadratic function in equation (10) and the non-linear function in equation (11), it follows from the graphical representation of their solutions that $C^{e*} < C^*$. That is, the optimal fractional level of the firm retained by the shareholder is less when stock options have incentive effects. Since C=1-c, it follows that the optimal level of executive stock options will be higher for the manager whose effort is motivated by the level of stock options, Type I manager. Furthermore, it is higher the greater the incentive effects of the stock options. This follows since with higher compensated effort supply elasticity, the vertical intercept of the asymptotic line shifts down thereby reducing the value of C^{e*} where the two lines intersect. Given the same value for z in both (11) or (10), the z produced by the Type I with the stronger incentive effects will suggest that a higher portion of this cash flow and firm value is due to managerial effort. This suggests that at time 1 the firm being divided up via the manager's exercise of his stock option has a portion coming from the manager's contribution to its size. On the other hand, the z produced by the Type II manager has no effort contribution. The Type II manager appears to be compensated for risk bearing alone. Thus it is not inconsistent that the level of stock option for this type of managers should be less than that for the manager whose efforts are affected by the stock option level.

COMPARATIVE STATIC ANALYSIS

Ideally one would wish that the operating parameters on which the Pareto optimal contract was based upon would remain the same throughout the contract period. In reality this is hard to achieve, given that there are events and market developments that will permanently change the operating parameters of the firm which are beyond the control of either the manager or the shareholder. These unexpected market developments can happen midstream of the duration of the compensation contract and change the firm's expected cash flow and volatility. Furthermore, risk attitudes generally change through time. As such, a long-term executive contract which awards executive stock options could easily be far from its optimal at some later time, thereby defeating the incentive effectiveness of the contract at the time it was signed. In the discussion following, the paper analyzes the direction and relative magnitude of adjustment to the initial pareto optimal contract when there are some perturbation in the value of some firm parameters, such as an increase in the expected firm cash flow, the volatility of the firm's random cash flows, and the risk attitude of the manager.

In other words, it addresses the sensitivity of the pareto optimal contract to parametric changes. Sensitivity of the contract, and consequently, its frequency of adjustment are related to the type of managers running the firm. The analysis below compares level of adjustment needed to restore pareto optimality or the relative degrees of sensitivity of contract between a Type I manager and a Type II manager.

The comparative analysis of the sensitvity of the optimal level of C to parametric changes is obtained by applying the Implicit Function Theorem (IFT) to the optimality conditions (11) and (10) (which are alternative expressions for equations (8) and (9), respectively). Comparing these results, one obtains an expression which tells us how the optimal stock option changes with perturbations in one of the parameter.[5] The shareholder can use these results to determine the direction and relative amount of adjustments to the level of executive stock option to maintain pareto optimality. Knowing the sensitivity of the contract enables the shareholder to determine the frequency of renegotiation or adjustment of long term executive contracts.

Sensitivity Of Contract To Changes In Risk Attitudes

Suppose that events and market conditions causes a permanent increase in *a*, the absolute risk aversion of the manager. This will result in the manager's assigning a higher risk premium for risk bearing. One solution is to modify the contract with more stock option with an accompanying decrease in the fixed salary, since it is assumed that c and w are substitutes. Another is to decrease the level of stock option and increase the fixed salary. It suggests that to maintain Pareto optimality, the optimal fraction of the firm which the manager receives through his stock options should decrease for either type of manager. Applying the Implicit Function Theorem (IFT) to the optimality conditions (10) and (11) and comparing the results using the same values for the other parameters in both expression:

Equation 12

$$\frac{dC}{da} = -\frac{F_a}{F_C} = \underbrace{\left\{\frac{V(1-C)(C+B)}{[2C(a+A)+aB+Ab-a]V}\right\}}_{\substack{\text{no incentive effects} \\ \text{Type II}}} > \underbrace{\left\{\frac{V(1-C)(C+B)}{[2C(a+A)+aB+Ab-a]V+\gamma}\right\}}_{\substack{\text{with incentive effects} \\ \text{Type I}}} > 0$$

where $b > B \geq 1$, $C \leq 1$ and $\gamma = \dfrac{e\varepsilon}{(1-C)^2}[-Bb+cB+Cb+C] + \dfrac{e\varepsilon}{(1-C)}[B+b+2z] > 0$.

(11) shows that an increase in the manager's risk aversion requires an increase in the ownership shares retained by the shareholder. In other words, an increase in the risk aversion of the manager will require a smaller amount of his income in the form of stock options. Type II managers will want less stock options as part of their compensation package as they become more risk averse. This comes as no surprise because as managerial risk aversion increases, the manager will prefer to have less of his income subject to uncertainty thereby increasing the relative proportion of his fixed salary, *w*. This increases the proportion of the manager's riskless income and reduces his risk bearing.

Relative to a Type II manager, the Type I manager needs smaller or less adjustments to his stock options for the same change in risk aversion because this manager can mitigate the effect of increased risk aversion on the risk premium, *caV*, by producing higher expected firm cash flow through increased effort, thereby restoring the the risk-return trade-off of his compensation package.

Sensitivity To Changes In The Level Of The Firm Expected Cash Flow

Firm parameters that are often monitored are performance measures associated with the firm's cash flow such as sales, net income and stock price. However, structural changes in the market can change permanently the distribution of the firm's cash flow and its expected value. As such, an increase in the expected value of the firm value while keeping the volatility constant will result in an increase in value of the manager's stock option due to the better change of higher cash flow in the good states. There is now a higher chance that his stock options will be in-the-money at expiration. For the Type II manager whose effort is invariant to the level of stock options so that stock options appears solely to serve as compensation for risk bearing, the adjustment to his package is obvious. To maintain pareto optimality, the level of the stock options has to be decreased. In so doing, the increased expected payoff to the manager is offset by the reduced number of stock options. This solution may not be too obvious for the Type I manager whose effort partly depends on the level of stock options, because decreasing the level of stock option may decrease his level of effort therefore defeating the effectives of stock options as an incentive tool to improve long run firm performance.

Applying the Implicit Function Theorem to the Pareto optimality conditions (10) and (11) and comparing the magnitude of their *absolute* values, equation (15) shows that the fraction of the firm retained by the shareholder, *C*, increases for both the Type I and Type II managers. However, the magnitude of the change in ownership retained by the shareholder is larger with a Type I manager and lesser with a Type II manager.

Equation 15

$$\frac{dC}{dz} = -\frac{F_z}{F_C} = -\underbrace{\frac{B-b}{2C(a+A)+aB+Ab-a}}_{\substack{\text{no incentive effects} \\ \text{Type II}}} > -\underbrace{\frac{B-b}{[2C(a+A)+aB+Ab-a]V+\gamma}}_{\substack{\text{with incentive effects} \\ \text{Type I}}} > 0$$

In terms of the manager's stock options, there will be less of a decrease in the contingent ownership claim of the Type I manager whose effort depends on stock options. Type II managers will experience a greater decrease in stock options than Type I managers for the same amount of increase in firm's expected cash flow *z*, because such an increase in *z* on the part of the Type II manager amounts to a windfall increase for firm cash flow, and consequently the value of his stokc options, with out the benefit of incentive effects or the supply of effort. On the other hand because Type I manager effort is affected by the level of stock options, dropping the level of stock options as much as that for a Type II manager can have an undesirable incentive effect on the level of effort that a Type I manager will expend to increase further firm cash flow. Hence the adjustment will not be as drastic as that for a Type II manager.

Assuming expected cash flows affect directly the market value of the firm, this inverse relationship between changes in the firm performance and the executive's total option value is consistent with the inititally puzzling empirical results found in Tables 6 and 8 of Murphy (1985) which suggest that the executive's total option value have a negative time series coefficient with stock price and sales.

Effect Of An Increase In The Volatility Of The Firm's Cash Flow

Consider the manager whose income comes from a state-independent fixed salary and a state contingent income, the stock option which is the risky component of his income. The manager can estimate his total expected income if the distribution of the firm's cash flow is given. The pareto optimal compensation package corresponds to a certain acceptable risk borne per unit expected income. Suppose expected cash flow is to remain the same, but V , the upper semi-variance of the firm's cash flow, increases due to some events in the market. This increase will result in the manager experiencing a higher risk borne per expected income. This decreases the manager's welfare and the original pareto optimal compensation package will no longer be optimal. Unlike an increase in risk attitude, an increase in the volatility of the firm's cash flow increases the risk premia of both the shareholder's, $(1-c)AV$, and that of the manager, cAV. Looking at equation (9), this statement is self-evident. For a Type II manager whose effort is invariant to stock options and for whom stock option serves as compensation for risk-bearing, one would guess that, if the volatility of firm cash flow increases due to some externalities and the manager is not motivated by the possibility of improved income from options holdings, the level of stock options should be increased to maintain pareto optimality. This can be verified by applying the Implicit Function Theorem to equation (10). It indicates that as the volatility of firm cash flow increases, the ownership retained by the shareholder should decrease.

Equation 13

$$\frac{dC}{dV} = -\frac{F_V}{F_C} = -\frac{-(B-b)z/V^2}{[2C(a+A)+aB+Ab-a]V} < 0 \qquad \text{Q.E.D.}$$

This suggests that additional stock options be provided to a Type II manager to compensate him for increased volatility of firm cash flow and for more risk bearing.

The opposite adjustment is needed for a manager whose effort increases with the level of stock options and with whom stock options have strong incentive effects. An increase in the volatility of earnings requires a decrease in the level of stock options to maintain pareto-optimality. Although the risk per unit expected return increases with increased firm volatiltiy, an increase in the volatility of the firms earnings does not necessary make him worst off, if he can maintain a certain risk return trade-off with the efforts expended to increase firm expected cash flows. This is true if the manager's effort supply elasticity is high such that $e\varepsilon(B+C)(b+C)/V^2 > -(B-b)/V^2$ or, the increase in income generated from effort compensates for the the incremental risk borne, thereby keeping his risk-return trade-off at the same level. This conclusion can readily be obtained by applying the IFT to equation (11) and realizing that an increase in the ownership retained by the shareholder translates to fewer stock options for the manager.

Equation 14

$$\frac{dC}{dV} = -\frac{F_V}{F_C} = -\frac{-(B-b)z/V^2 - e\varepsilon(B+C)(b+C)/V^2}{[2C(a+A)+aB+Ab-a]V+\gamma} > 0 \qquad \text{Q.E.D.}$$

To maintain proper incentives under volatile market conditions, this result suggests that the shareholder appears better off hiring managers motivated by contingent share ownership via stock options.

In both cases, we let $b > B$ so that it follows that the shareholder's preference for favorable states of nature, or for income in these high states, is relatively greater than that of the manager. This is probably not an unrealistic assumptions, because shareholders as principal owners of the firm should be inherently more interested in the favorable performance of the firm than their agents, the managers.

CONCLUSION

The analysis of executive stock options provided in this paper limits itself to a principal-agent framework when hedging and trading of executive stock options are restricted during the contract period. The closed form solutions for the optimal contract comprise a compensation package with only a fixed salary and stock options. Not surprisingly, we find that the optimal level of executive stock options is greater for managers motivated by the incentive effects of stock options. For managers whose effort is invariant to the level of their stock options, the sole function of options is risk bearing and the level of stock options is less.

While the shareholder's fraction of ownership retained will be lower when they hire managers who are motivated by stock options, the shareholder can benefit by having a less volatile ownership claim. This effect is especially relevant if we have shareholders who are concerned over the certainty of firm ownership and control for firms operating in dynamic environments with uncertainties about firm cash flows and risk attitudes. An interesting result which follows from the analysis is that when the volatility in the firm's cash flow increases, pareto optimality in the compensation contract for a manager with incentive effects requires a decrease in his stock options. This increases the fractional ownership of the shareholder. Whereas for a manager with no incentive effects, an increase in the volatility of the firm's cash flow requires an increase in stock options.

Another interesting result is that compensation contracts with stock options stay near pareto optimality. Shareholders benefit from this in two ways. First, shareholder wealth increases since managerial incentives remain continuously aligned with those of their owners. Secondly, shareholders avoid the costs associated with frequent recontracting with managers. Similarly, for either changes in managerial risk aversion or expected firm cash flows, the adjustment to maintain a pareto optimal compensation contract is smaller if stock options are present. These are true for contracts drawn with managers whose efforts are motivated by stock options.

Thus while these manager motivated by stock options (Type I) inititally requires a higher level of stock options than other managers, there are three benefits to the shareholder to hiring them. One, the shareholder's ownership is less volatile in a highly dynamic environment when parameters are constantly changing. Two, under certain conditions, the shareholder's retained ownership may actually be higher than what he anticipated. Three, if stock options motivates positive effort supply elasticity, the value of the firm increases and, figuratively speaking, the shareholder ends up retaining, with less uncertainty, a larger share of a bigger pie.

ENDNOTES

1. At the border: $i_h(\theta^0) = (1-c)w + c(\theta^0 - X)$. Since $\theta^0 = X+w$, it follows that $i_h(\theta^0) = (1-c)w + c(X+w-X) = w$.

2. These two expressions have been considerably simplified by applying the first order condition, $E_l[u_e]+E_h[u_e] + E_h[u_i c] = 0$.

3. By replacing c by 1-C, we are able to express the solution in terms of familiar functional forms and thus facilitate our analysis.

4. Since both shareholder can have only long positions in the firm, we disregard solution with negative values of C or c.

5. Another way to determine the change in the optimal level of C when stock options have incentive effects is to trace the shift of the point of intersection of the two lines (given by the LHS and RHS of (11) when a parameter is changed. This can be illustrated by using Figure 1, as well. For the case when there are no incentive effects, one needs only to note the change in the positive horizontal intercept of the parabola given be (10).

6. $E_h[\theta(\theta - \bar{\theta}_h)] = E_h[(\theta - \bar{\theta}_h)(\theta - \bar{\theta}_h)] + \bar{\theta}_h E_h[(\theta - \bar{\theta}_h)] = E_h[(\theta - \theta_h)^2] + 0$

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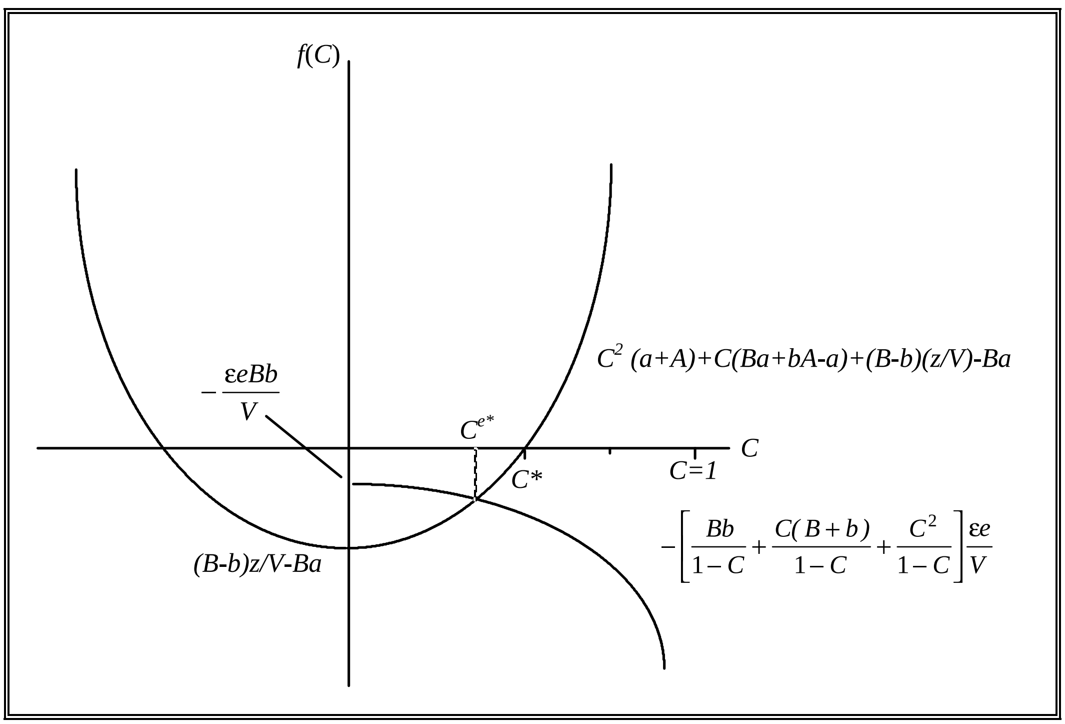
FIGURE 1
The Optimal Level When There Are Incentive Effects, C^{e*},
Is The Intersection Of The Two Non-Linear Functions



APPENDIX

Proof to Proposition 1

Forming the Lagrangean for the shareholder's maximization problem with a Type I manager.

Equation a.1

$$L = H(w,c) + \lambda(u^* - R(w,c))$$

Its first order conditions are:

Equation a.2

$$\frac{\partial L}{\partial c} = E_l[U_I e_c] + E_h[(1\text{-}c)U_I e_c - E_h[(\theta + e\text{-}w\text{-}X)U_I] + \lambda\{-E_l[u_e e_c] - E_h[(\theta + e\text{-}w\text{-}X)u_i - E_h[u_e e_c] - E_h[cu_i e_c]\} = 0$$

Equation a.3

$$\frac{\partial L}{\partial w} = E_l[U_I e_w] + E_l[-U_I] + E_h[-(1\text{-}c)U_I + E_h[(1\text{-}c)U_I e_w] + \lambda\{-E_l[u_e e_w] - E_l[u_i] - E_h[(1\text{-}c)u_i] - E_h[u_e e_w] - E_h[cu_i e_w]\} = 0$$

Applying the Envelope Theorem we can simplify equations (a.2) and (a.3) to

Equation a.4

$$\frac{\partial L}{\partial c} = \{E_l[U_I] + E_h[(1\text{-}c)U_I]\}e_c - E_h[(\theta + e\text{-}w\text{-}X)U_I] - \lambda E_h[(\theta + e\text{-}w\text{-}X)u_i] = 0$$

Equation a.5

$$\frac{\partial L}{\partial w} = \{E_l[U_I] + E_h[(1\text{-}c)U_I\}e_w - E_l[U_I] - E_h[(1\text{-}c)U_I] - \lambda\{E_l[u_i] + E_h[(1\text{-}c)u_i]\} = 0$$

Dividing equation (a.4) by equation (a.5) yields:

Equation a.6

$$\frac{\{E_l[U_I] + E_h[(1-c)U_I]\}e_c - E_h[(\theta + e - w - X)U_I]}{E_l[U_I] + E_h[(1-c)U_I\}e_w - E_l[U_I] - E_h[(1-c)U_I]} = \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}$$

Rearranging terms:

Equation a.7

$$\{E_l[U_I] + E_h[(1\text{-}c)U_I]\}e_c - E_h[(\theta + e\text{-}w\text{-}X)U_I] = \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}\{E_l[U_I] + E_h[(1\text{-}c)U_I\}e_w + \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}\{- E_l[U_I] - E_h[(1\text{-}c)U_I])\}$$

and dividing by $\{E_l[U_I]+E_h[(1\text{-}c)U_I]\}$

Equation a.8

$$e_c - \frac{E_h[(\theta + e - w - X)U_I]}{E_l[U_I] + E_h[(1-c)U_I]} = \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}(e_w - 1)$$

Hence,

Equation a.9

$$e_c - \left(\frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}\right)e_w = \frac{E_h[(\theta + e - w - X)U_I]}{E_l[U_I] + E_h[(1-c)U_I]} - \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}$$

Applying Theorem 1 to the LHS, we obtain:

Equation a.10

$$\frac{e\varepsilon}{c} = \frac{E_h[(\theta + e - w - X)U_I]}{E_l[U_I] + E_h[(1-c)U_I]} - \frac{E_h[(\theta + e - w - X)u_i]}{E_l[u_i] + E_h[(1-c)u_i]}$$

where the first expression (actually, its inverse) on the RHS upon dividing and multipying by $E_h\ [U_I]$ becomes:

Equation a.11

$$\frac{E_l[U_I] + E_h[(1-c)U_I]}{E_h[U_I(\theta + e - w - X]} = \frac{E_l[U_I]/E_h[U_I] + (1-c)}{\frac{E_h[\theta U_I]}{E_h[U_I]} + e - w - X}$$

Applying a Taylor expansion to $\frac{E_h[\theta U_I]}{E_h[U_I]}$ about $\bar{\theta}$ we obtain:

Equation a.12

$$\frac{E_l[U_I] + E_h[(1-c)U_I]}{E_h[(\theta + e - w - X)U_I]} = \frac{B + (1-c)}{\bar{z} - (1-c)AV_h}$$

These are parameters associated with the shareholder. $V_h = E_h[(\theta - \vartheta_h)^2]$ is the measure of uncertainty of firm output and his income in high income state h; $A \equiv -U_{II}(\mathrm{I}_h(\theta))/U_I(\mathrm{I}_h(\theta))$, the shareholders' absolute risk aversion; $\bar{z} = \vartheta_h + e\text{-}w\text{-}X$ is the mean net gain in the high income state, unadjusted for risk and ownership sturcture; and $B \equiv E_l[U_I]/E_h[U_I]$ is a measure associated with the shareholders preference for ownership income.

The parameters associated with the manager can likewise be obtained by multiplying and dividing the second term in equation (a.10) by $E_h[u_i]$, and applying a Taylor expansion about θ to obtain:

Equation a.13

$$\frac{E_l[u_i] + E_h[(1-c)u_i]}{E_h[(\theta + e - w - X)u_i]} = \frac{b + (1-c)}{\bar{z} - caV_h}$$

where $a \equiv -u_{ii}(\mathrm{i}_h(\theta))/u_i(\mathrm{i}_h(\theta))$ is the manager's absolute risk aversion at high incomes and $b \equiv \pi_l u_i/E_h[u_i]$ is associated with the manager's preference for ownership income.

Applying the results in equations (a.12) and (a.13) into equation (a.10), we obtain an equation for the optimal level of c, when performance pay has an incentive effect on the the level of effort.

Equation a.14

$$\frac{e\varepsilon}{c} = \frac{\bar{z} - (1-c)AV_h}{B + (1-c)} - \frac{\bar{z} - caV_h}{b + (1-c)}$$

Q.E.D.

Proof to Proposition 2

The lagrangean of the shareholder maximization problem with a Type II manager:

Equation a.15

$$L = H(w,c) + \lambda(u^{*} - R(w,c))$$

with the first order conditions being:

Equation a.16

$$\frac{\partial L}{\partial w} = -E_l[U_I] - E_h[(1-c)U_I] + \lambda\{-\pi_l u_i - E_h[(1-c)u_i]\} = 0$$

and

Equation a.17

$$\frac{\partial L}{\partial w} = -E_h[(\theta-w-X)U_I] + \lambda(-E_h[(\theta-X-w)u_i]) = 0$$

Eliminating λ by dividing equation (a.16) by equation (a.17) we obtain a measure for the shareholder's and manager's marginal rate of substition given by:

Equation a.18

$$\frac{E_l\,[U_I\,] + (1-c)E_h\,[U_I\,]}{E_h\,[(\theta - w - X)U_I\,]} = \frac{\pi_l u_i + (1-c)E_h\,[u_i\,]}{E_h\,[(\theta - X - w)u_i\,]}$$

where the RHS (LHS) gives the manager's (shareholder's) expected marginal utility of income in all state to expected marginal utility of income in the high state, given the contractual arrangement.

Dividing and multiplying the LHS by $E_h\,[U_I]$, and the RHS by $E_h[u_i]$, and using the identitiy $E_h[(\theta\text{-}w\text{-}X)U_I] \equiv E_h[\theta U_I]\text{-}wU_I\text{-}XU_I$, we obtain:

Equation a.19

$$\frac{E_l\,[U_I\,] / E_h\,[U_I\,] + (1-c)}{E_h\,[\theta U_I\,] / E_h\,[U_I\,] - w - X} = \frac{\pi_l u_i / E_h\,[u_i\,] + (1-c)}{E_h\,[\theta u_i\,] / E_h\,[u_i\,] - w - X}$$

The expressions $\frac{E_h\,[\theta U_I\,]}{E_h\,[U_I\,]}$ and $\frac{E_h\,[\theta u_i\,]}{E_h\,[u_i\,]}$ in the denominator of equation (a.19) can be simplified to more familiar economic terms as follows:

Using a Taylor expansion of $U_I \equiv U_I(I_h(\theta))$ about $\overline{\theta}_h$

Equation a.20

$$U_I\,[I_h(\theta)] = U_I[I_h(\overline{\theta}\,)] + (\theta - \overline{\theta}_h\,)(1-c)U_{II}[I_h(\overline{\theta}\,)]$$

where $\overline{\theta}_h = E_h[\theta]$ and $I_h(\overline{\theta}\,) \equiv (1-c)\,\overline{\theta}_h + cX - (1-c)w$ is the shareholder's mean income in high states.

Applying the expectation operator $E_h[.]$ to both sides and using the fact that $E_h[\theta - \theta_h\,] = 0$, equation (a.20) becomes:

Equation a.21

$$E_h[U_I] = U_I(I_h(\overline{\theta}\,))$$

Multiplying equation (a.20) by θ, and then applying the expectation operator to both sides, we get:

Equation a.22

$$E_h[\theta U_I] = E_h[\theta U_I(I_h(\bar{\theta}))] + E_h[\theta(\theta-\bar{\theta}_h)(1-c)U_{II}(I_h(\bar{\theta}))]$$

Upon dividing by equation (a.21), we obtain:

Equation a.23

$$\frac{E_h[\theta U_I]}{E_h[U_I]} = \bar{\theta}_h + E_h[\theta(\theta-\bar{\theta}_h)](1-c)\frac{U_{II}(I_h(\bar{\theta}))}{U_I(I_h(\bar{\theta}))}$$

where $E_h[\theta(\theta-\bar{\theta}_h)] = E_h[(\theta-\bar{\theta}_h)^2]$ is the variance[6] of the high outcome states which we will denote by V_h. This measures the degree of uncertainty of income in state *h*. $-\frac{U_{II}(I_h(\bar{\theta}))}{U_I(I_h(\bar{\theta}))} = A$ is the shareholder's absolute risk aversion for state *h* income.
Thus, equation (a.23) becomes:

Equation a.24

$$\frac{E_h[\theta U_I]}{E_h[U_I]} = \bar{\theta}_h - (1-c)AV_h$$

Since θ is always greater than zero in state *h* and the marginal utility of income is always positive, it follows that the RHS of equation (a.24) is also greater than zero, or:

Equation a.25

$$\bar{\theta}_h - (1-c)AV_h > 0$$

Following a similar approach, $\frac{E_h[\theta u_i]}{E_h[u_i]}$ from equation (a.19), associated with the manager's utility in the high income states, can be shown to be:

Equation a.26

$$\frac{E_h[\theta u_i]}{E_h[u_i]} = \bar{\theta}_h - caV_h > 0$$

where $a = -\frac{u_{ii}(i_h(\bar{\theta}))}{u_i(i_h(\bar{\theta}))}$ is the manager's absolute risk aversion associated with state *h* income.

Substituting equations (a.26) and (a.24) into equation (a.19) and rearranging terms, we obtain a closed form solution determining the optimal level of *c*:

Equation a.27

$$\frac{\bar{\theta}_h - w - X - (1-c)AV_h}{E_l[U_I]/E_h[U_I] + 1 - c} = \frac{\bar{\theta}_h - w - X - caV_h}{\pi_l u_i / E_h[u_i] + 1 - c} \qquad \text{Q.E.D}$$

This is a quadratic equation in *c* where the LHS contains parameters relating to the shareholder while the RHS contains parameters relating to the manager.