INFLATION, CAPITAL STRUCTURE, AND IMMUNIZATION OF THE FIRM'S GROWTH POTENTIAL

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Abstract

In this present study we develop a general case model of corporate growth potential. This model enables us to determine the conditions which must prevail in order for neither inflation in input and/or output prices nor changes in interest rates to have an impact on the firm's growth potential and hence its investment decision. We find that immunizing the firm's real growth potential against the effects of inflation and interest rate fluctuations generally requires frequent changes in its capital structure, which may be a costly strategy to implement.

INTRODUCTION

Myers and Majluf [16] develop a model in which a firm must issue common stock to raise cash to take advantage of a valuable investment opportunity, and the firm's managers have more information about the valuable investment opportunity than do potential investors. The model shows that because of information asymmetry the firm may pass up the valuable investment opportunity. The model also shows that it is better to rely on internal sources of funds and to carry sufficient financial slack to respond to good investment opportunities as they arise.

The conclusions of the model of Myers and Majluf [16] are particularly relevant to small but growing firms operating in high technology industries. Investment in new, valuable research and development may be difficult to finance by issuing common shares, since the fair valuation of these new shares is hampered by the hazards associated with financing such investments without having access to detailed information about them. Small but growing high technology firms may not be able to reduce this information asymmetry, since revealing the information required by investors may enable competitors to adopt or imitate the new technologies at a small marginal cost. In fact, this transaction cost problem may be the reason many small but high technology firms with promising R&D investments approach large technology firms with proposals of equity stakes that often lead to a seat on the board of directors.

Based on the above discussion, it is very clear that small but growing firms operating in high technology industries are better off having a strategic financial management process in place, with the goal of accumulating financial slack, which enables them to make as many valuable investments as possible. An integral aspect of this strategic financial management process must be minimizing or hedging against the risks that the firm will have to assume while achieving the objective of accumulating financial slack. If the concept of risk is broadened beyond the shareholder perspective to include the factors of uncertainty with which managers typically have to cope, two critical factors can be identified: 1) the differential rate of inflation between product prices and factor input costs, which affects the firm's profit margin and real growth in financial slack; and 2) the exposure of the firm's profitability and financial slack growth prospects to interest rate fluctuations.

Inflation distorts the firm's net profitability and its ability to accumulate financial slack for many reasons. If LIFO accounting is used, cost of goods sold is charged against sales at current production costs, which are higher, resulting in lower reported profit margins. In addition, higher factor input prices lead to a gap between historical depreciation charges, which are tax deductible, and the cost of replacing fixed assets. Moreover, the firm's interest expense will increase as a response to increasing price levels, resulting in a decline in taxable income.

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Furthermore, if the firm is unable to pass on increases in production costs through similar increases in product prices, the firm's gross margins will be adversely affected.

Interest rate fluctuations can also have major effects on the firm's profitability and financial slack growth prospects. As the firm's interest expense on existing debts increases, taxable income decreases. Yet, a firm's potential to grow can be unaffected for a certain period of time if interest rates rise but the firm's cost of debt does not rise simultaneously because the firm's debt contracts have fixed interest rates. The percentage of the firm's debt contracts that have interest rates tied to certain market rates will determine to what extent the firm's profitability and its prospects for growth in financial slack are exposed to interest rate fluctuations.

Lewellen and Kracaw [14] develop a model for the sustainable growth rate of a firm in which they assume that the firm's operating milieu is one where product prices and factor input costs rise and fall at the same annual rates. The model shows that a firm can maintain a certain constant debt ratio that immunizes its growth potential (i.e. financial slack accumulation) against the effects of inflation. Accordingly, the capital structure policy of a firm is an important component of its financial management strategy, as it can eliminate some of the risks that the firm assumes while pursuing the object of accumulating financial slack.

Research Objective

Our objective in this paper is to examine whether relaxing the assumption of Lewellen and Kracaw [14] regarding the equality of inflation in input and output prices alters their conclusion that there is a firm-specific debt ratio which insulates that firm's sustainable growth against the effects of inflation. In developing our corporate growth model, we also relax the assumptions that all the firm's debt has a freely floating interest rate and that interest rates respond to price level changes following the Fisher effect or the Darby effect [4].

The paper is organized as follows: Section II develops the corporate growth model and shows that it is the general case of many of the well-publicized models found in the literature. Section III determines the strategy based on the model that enables management to preserve its sustainable growth and financial slack accumulation against fluctuating inflation and/or interest rates. Section IV concludes the paper with a summary of the implications of our analyses and some suggestions for further research.

A MODEL FOR CORPORATE GROWTH POTENTIAL

Assumptions

To develop the model we start by making a set of assumptions about the response of various assets and liabilities to rising sales volume in an inflationary environment. The first assumption is that cash needs and net receivables will automatically increase at the same rate as do nominal sales, even with a change in both unit volume and unit prices [8, 9, 14]. Therefore, cash and net receivables will increase with sales at the rate (C)(dS/dt), where C is the ratio of cash and net receivables to annual sales and dS/dt is the firm's instantaneous sales volume growth rate.

Equation 1

Increase In Cash And Net Receivables = (C)(dS/dt)

The second assumption is that spontaneous liabilities, i.e. accounts payable, accrued payroll, unearned revenue, and other accruals, will also increase at the same rate as do nominal sales [6, 9, 14]. Thus, the required rate of increase in spontaneous liabilities will be (L)(dS/dt), where L is the ratio of spontaneous liabilities to annual sales.

Equation 2

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Increase In Spontaneous Liabilities = (L)(dS/dt)
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The third assumption is that the firm takes advantage of the tax benefits of LIFO accounting. Accordingly, inventory will increase at a rate less than that of nominal sales, this rate being the full rate of nominal sales less the avoided inflation component on the already-in-place inventory base [14]. That is, inventory will increase at the instantaneous rate of:

Equation 3

Increase In Inventory, d(INV)/dt = I(dS/dt) - jIS

where *I* is the ratio of inventory to annual sales and *j* is the annual instantaneous *percentage* rate of change in the firm's factor input prices.

In addition, we assume that the book depreciation schedule is initially set by management to match the *expected* physical deterioration pattern. The uncertainty, however, is due to the possibility of emergence of new *unexpected* production processes or product attributes that render existing fixed assets obsolete. Moreover, book fixed assets will increase less rapidly than nominal sales in an inflationary environment [8, 14]. Specifically, the instantaneous growth rate of net fixed assets will be the rate of sales growth rate *less* the avoided inflation component on the already-in-place fixed assets *plus* the incurred inflation component on the replacement of the expected and unexpected depreciation of the existing fixed asset base. Accordingly, if we denote the ratio of net fixed assets to annual sales by *F*, the annual instantaneous *expected* percentage depreciation rate by *k* and the annual instantaneous *unexpected* percentage depreciation rate by *u*, the instantaneous actual dollar depreciation rate will be (k+u)FS. The gap between historical and replacement costs will be j(k+u)FS and the instantaneous rate of increase in net fixed assets will be:

Equation 4

$$\frac{d(FIXED)}{dt} = \frac{F(dS}{dt}) - \frac{jFS}{f} + \frac{j(k+u)}{FS} = \frac{F(dS}{dt}) - \frac{(jFS)(1-k-u)}{f}$$

We also postulate that firms *usually* raise debt capital from two major sources: revolving credit lines which are usually equal to a fixed percentage of working capital, and secured debt against which fixed assets would be pledged as collateral. The secured debt is usually a certain percentage of the value of the fixed assets. In addition, it can be said that bond covenants usually include restrictions on the disposition of the firm's assets, restrictions on the issuance of new debt, and some requirements to maintain the firm's working capital above a certain minimum level [17]. All this leads us to conclude that new financing can be assumed to be based on the market value of the firm's new assets. Specifically, we model the instantaneous growth rate of total debt as:

Equation 5

$$\frac{d(Debt)}{dt} = \beta_1(C+I-L)(\frac{dS}{dt}) + \beta_2(F)(\frac{dS}{dt})$$

where:

(C+I-L)(dS/dt)	represents the instantaneous dollar increase in working capital
(F)(dS/dt)	represents the instantaneous dollar increase in net fixed assets; and
β_1 and β_2	are the percentages of new working capital and new net fixed assets, respectively,
	against which the firm can borrow equal amounts of funds.

It is well known that the dividend decisions of leveraged firms are usually constrained by some covenants in the bond indentures. Kalay [11] examined a random sample of 150 firms and found that the dividend constraint limits all forms of payments and does not distinguish between cash dividends and share repurchases. In addition to the study by Kalay [11], theoretical and empirical work on dividend signaling [1, 2, 3, 5, 10, 15] confirms that the announcement of a dividend cut often leads to a drop in the firm's stock price, whereas the announcement of a dividend increase often leads to an increase in the firm's stock price. Therefore, the direct legal constraint on dividend payments coupled with the information asymmetry and signaling context of the firm, as well as the

simultaneous need for management to keep cash outflows at a minimum in order to finance growth, while maintaining the highest and most stable stock price, all lead us to consider that dividend payments will be constant over the planning horizon and that the company will not resort to the issuance of new equity. However, these constant dividend payments are not constrained to be cash payments. They can be any combination of cash payments and/or share repurchases. We denote the instantaneous dollar amount of dividends by D.

Equation 6

The firm's total annual dividend payments = $\int_{0}^{t=1} D(t) dt$

We postulate that product prices and production costs are rising at two different percentage rates, i and j respectively. Product prices and factor input costs rise and fall at two different and separate percentage rates as a result of the frequent changes in the output and input market structures. These changes are, for instance, due to the shifts in the bargaining power of suppliers or buyers in that market, or in the rivalries between the existing firms in the industry, and will affect the firm's net profit margin. The firm's cost of goods sold charged against sales at current production costs will be higher if LIFO accounting is used. In addition, increasing factor input prices will result in a gap between historical depreciation charges that are tax deductible and the cost of replacing fixed assets. Moreover, the firm's interest expense will increase as a response to increasing price levels, by a magnitude following the Fisher affect, the Darby effect [4] or anything in between. This increase results in a decline in reportable net income.

To quantify the effects of all the above-mentioned factors on the firm's net profit margin *m*, we note that with product prices and factor input costs rising at two different rates, the firm's gross profit margin will change instantaneously by *approximately* (*i*-*j*) (See Appendix A). In addition, as a result of the firm's cost of debt adjusting to inflation, the instantaneous rate of change in the firm's interest expense will be equal to the inflation adjusted term *e* times the already existing debt base that has a freely floating interest rate, being the portion α of all the firm's outstanding debt base. The symbol *e* is the adjustment in interest rates due to inflation. It could be equal to the Fisher Effect or the Darby Effect [4] or something else. Therefore,

Equation 7

 $d(INT)/dt = (e)(\alpha)[\beta_1(C+I-L)+\beta_2(F)](S)$

Finally, the annual depreciation rate that was reflected in the pre-inflation profit margin m will fall in relation to increasing nominal sales, resulting in higher reported profits as percentage of sales. If we denote the corporate tax rate by T, all these effects will cause the rate at which the firm's equity base is being augmented to become:

Equation 8

$$d(EQUITY)/dt = mS + (1-T)(i-j)(S) - (1-T)(e)(\alpha)[\beta_1(C+I-L)+\beta_2(F)](S) + (1-T)(jFS)(k+u) - D$$

The Model

Having determined the responses of various assets and liabilities to rising sales volume, we set the following condition for the firm's instantaneous growth:

Instantaneous Uses Of Funds = Instantaneous Sources Of Funds

The above condition implies that:

Equations [1+3+4] = Equations [2+5+8]

Substituting for the above equations we get:

Equation 9

$$\begin{aligned} (C)(dS/dt) + (I)(dS/dt) - (jIS) + (F)(dS/dt) - (jFS)(1-k-u) \\ &= (L)(dS/dt) + [\beta_1(C+I-L) + \beta_2(F)](dS/dt) + mS + (1-T)(i-j)(S) \\ &- (1-T)(e)(\alpha)[\beta_1(C+I-L) + \beta_2(F)](S) + (1-T)(jFS)(k+u) - D \end{aligned}$$

Consolidating and dividing through by *S*, we get the following *nominal* instantaneous equilibrium growth rate model for the firm:

Equation 10

$$dS / dt = g^* = \frac{mS - D + i(1 - T)S + j[I + F - T(k + u)F - (1 - T)]S - (e)(\alpha)(1 - T)[\beta_1(C + I - L) + \beta_2(F)](S)}{[(1 - \beta_1)(C + I - L) + (1 - \beta_2)(F)](S)}$$

where:

- g^* is the firm's nominal instantaneous equilibrium growth rate;
- *m* is the firm's net profit margin;
- S is the firm's annual sales;
- C is the ratio of cash and net receivables to annual sales;
- L is the ratio of spontaneous liabilities to annual sales;
- *I* is the ratio of inventory to annual sales;
- F is the ratio of net fixed assets to annual sales;
- k is the annual instantaneous expected depreciation rate;
- u is the annual instantaneous unexpected depreciation rate;
- β_1 percentage of new working capital against which the firm can borrow;
- β_2 percentage of new fixed assets against which the firm can borrow;
- D is the firm's instantaneous amount of dividends;
- T is the firm's tax rate;
- α is the percentage of all the firm's outstanding debt base that has a freely floating interest rate;
- *i* is the annual instantaneous inflation rate in the firm's product prices;
- *j* is the annual instantaneous inflation rate in the firm's factor input prices;
- e is the adjustment in interest rates due to inflation or any other specified factor of interest.

The Generality Of The Model

A review of the literature in the area of sustainable growth reveals many competing models (See Appendix B). The new model that is developed in this paper can be shown to be the general case of many of the models discussed in the literature. In fact, our model would be similar to the real sustainable growth rate model of Lewellen and Kracaw [14] if $\beta_1=\beta_2$, i=j=e=0 (i.e. there is no inflation and interest rates are stable), and *D* is a constant percentage of net income. Similarly, the nominal sustainable growth rate models of Lewellen and Kracaw [14] are also special cases of our model since they assume that i=j, $\beta_1=\beta_2$, $\alpha=1$, and e=i for the model with Fisher's Effect, and e=i/(1-T) for the model with Darby's Effect. Moreover, the Ulrich and Arlow [18] model becomes a special case of our model if one assumes that $\beta_1=\beta_2=\beta$, i=j=e=0, *D* is a constant percentage of net income, and borrowed funds are equal to $\beta(C+I-L+F)S$. Furthermore, since the models of Kisor [12], Lerner and Carleton [13], Higgins [7] and Ulrich and Arlow [18] can all be shown to be versions of the same conceptualization of corporate growth (see Appendix C), they are also special cases of our model.

INFLATION AND CORPORATE GROWTH POTENTIAL

The corporate growth model developed above enables us to identify the conditions which must prevail in order for either inflation in input and/or output prices, or changes in interest rates *not* to impact the firm's growth potential and accumulation of financial slack. For that purpose, our model can be rearranged as follows:

Equation 11

$$g^* = \frac{mS - D}{[(I - \beta_1)(C + I - L) + (I - \beta_2)(F)](S)} + \frac{i(1 - T)(S)}{[(I - \beta_1)(C + I - L) + (I - \beta_2)(F)](S)}$$

$$+\frac{j[I+F-T(k+u)F-(I-T)]S}{[(I-\beta_{1})(C+I-L) + (I-\beta_{2})(F)](S)} - \frac{(e)(\alpha)(I-T)[\beta I(C+I-L) + \beta 2(F)](S)}{[(I-\beta_{1})(C+I-L) + (I-\beta_{2})(F)](S)}$$

For the case where i=j=e=0, the above model simplifies to the *real* instantaneous equilibrium growth rate model for the firm g_r^* :

Equation 12

$$g^{*}_{r} = \frac{mS - D}{[(I - \beta_{I})(C + I - L) + (I - \beta_{2})(F)](S)}$$

Therefore, our nominal growth model can be simplified as follows:

Equation 13

$$g^{*} = g^{*}_{r} + \frac{i(1-T)}{[(1-\beta_{1})(C+I-L) + (1-\beta_{2})(F)]} + \frac{j[I+F-T(k+u)F-(1-T)]S}{[(1-\beta_{1})(C+I-L) + (1-\beta_{2})(F)]} - \frac{(e)(\alpha)(1-T)[\beta_{1}(C+I-L) + \beta_{2}(F)]}{[(1-\beta_{1})(C+I-L) + (1-\beta_{2})(F)]}$$

For the *nominal* equilibrium growth rate to be equal to the *real* equilibrium growth rate *plus* the inflation rate, i.e. $g^* = g^*_r + i$, the following two conditions must prevail:

Equation 14

$$\frac{i(1-T) + j[I+F-T(k+u)F-(1-T)]}{[(1-\beta_1)(C+I-L) + (1-\beta_2)(F)]} = i$$

Equation 15

$$\frac{(e)(\alpha)(1-T)[\beta_1(C+I-L)+\beta_2(F)]}{[(1-\beta_1)(C+I-L)+(1-\beta_2)(F)]} = 0$$

Assuming for simplicity that $\beta = \beta_1 = \beta_2$, condition (14) would be met if:

Equation 16

$$\beta = 1 - \frac{(1-T)}{(C+I-L+F)} + \frac{j[I+F-T(k+u)F-(I-T)]}{i(C+I-L+F)}$$

and condition (15) would be met if:

Equation 17

 $\alpha = 0$

Accordingly, the firm can immunize its *real* growth potential from the effects of inflation (or deflation) and interest rate fluctuations by maintaining a debt ratio as given by equation 16, and converting all of its debt contracts with floating interest rates into fixed rate contracts, i.e. adhering to equation 17. This will result in a stable after-tax profit margin on sales. In addition, if dividends are assumed to be a constant percentage of the firm's after-tax earnings, meeting the conditions set by equations 16 and 17 will lead to stable nominal dividend growth of g^* , and payouts in real terms to existing shareholders matching those that would have occurred in the absence of inflation.

The debt ratio corresponding to equation 16 is a function of the firm's real operating parameters as well as the inflation rates in input and output prices. This finding implies that in order to immunize both its real growth potential and financial slack accumulation from the effects of inflation (or deflation) and interest rate fluctuations, a firm will have to relax its financing policy and adjust its debt ratio every time the inflation rate changes, except in the following special cases:

Equation 18

$$i = j$$
; or

Equation 19

$$(k+u) = \frac{I+F-(1-T)}{TF}$$
 or equivalently
$$T = \frac{I+F-1}{(k+u)F-1}$$

Equation 20

j = 0

In any one of the above special cases the firm can immunize its real growth potential by adjusting its capital structure only once. This is consistent with the Lewellen and Kracaw [14] analysis that assumed i = j, i.e., inflation in output prices equal to inflation in input prices. However, our analysis, which is based on the assumption that the inflation rate for factor inputs is different from the inflation rate for the firm's products, clearly shows that in *general* the firm will have to adjust its debt ratio more often. Moreover, equation 16 and 17 imply that the effects of inflation on a firm's growth prospects and hence its investment decision is *not independent* of its capital structure since the debt ratio is part of these two conditions.

Adjusting the debt ratio frequently in order to continuously immunize the firm's growth potential and financial slack accumulation against inflation and interest rate changes may be a difficult and costly strategy to implement. Restrictive covenants in bond indentures may restrict management's ability to do so. In addition, adjusting the debt ratio has signaling implications in the information asymmetry milieu of the firm. Accordingly, the benefits of preserving the firm's ability to undertake all profitable investments as they arise must be considered in light of the costs of minimizing the risks assumed in achieving that strategic financial management objective.

We have assumed that the interest rates that firms must pay in an inflationary environment rise immediately from their low pre-inflation levels. With $\alpha = 0$, i.e. all of the firm's debt being fixed rate in nature, there would be a period of time–until the debt matured–during which the firm's capacity to acquire additional real assets or to distribute cash to shareholders would be temporarily unaffected. On the other hand, if that fixed rate debt was

incurred in a high-inflation setting, and the rate of inflation has subsequently gone down, then this would constitute a lost opportunity to save on interest costs. In that case, an $\alpha = 1$, i.e. all the firm's debt being variable, would represent the condition that enables the firm to acquire additional real assets and/or distribute higher dividends to shareholders. However, for a firm to have all variable debt linked to market interest rates is risky and requires hedging strategies such as interest rate caps to guard against the risk of default from higher interest rates.

CONCLUSION

This paper develops a corporate growth rate model that incorporates the major determinants of the firm's value and the major risks with which management has to cope. It is shown that the model is a useful strategic financial management tool for formulating a financing strategy that immunizes the firm's growth potential and hence its investment decision against fluctuating inflation and/or interest rates. Having such a strategic financial management objective is particularly important for small but growing firms operating in high technology industries, as these firms may otherwise be forced to pass up valuable investment opportunities.

The main conclusion of our model is that in order to immunize the firm's growth potential and hence its investment decision against the effects of inflation and interest rate fluctuations, management will have to make frequent adjustments to the capital structure. This conclusion is in sharp contrast with the conclusion of Lewellen and Kracaw [14], that a firm has to choose a certain debt ratio only once in order to achieve immunization permanently. The main reason for our different conclusion is that we relax their restrictive assumption of equal inflation rates in output and factor input markets.

Suggestions For Future Research

A fruitful avenue of research based on the current model would be in the area of game-theoretic modeling. By setting some distributional and behavioral assumptions for the factors determining the growth rate of a hypothetical firm in a hypothetical industry, a simulation exercise could be used to: 1) investigate and quantify the tradeoffs facing the firm while trying to maintain its equilibrium growth rate; 2) examine the impact of the firm's equilibrium growth rate on the industry growth rate structure, and concentration; and 3) observe the characteristics of the fastest and slowest growing firms in that hypothetical industry. The game-theoretic model could be made even more challenging and realistic by establishing entry and exit barriers to the industry, and assuming some cost and pricing dynamics with learning diffusing across firm boundaries. Such a game-theoretic model would provide new insights about firm operating decisions and how they impact the overall position of the firm in its industry.

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APPENDIX A

Sales = (Units) (Unit Price) = (s)(P) and

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Total Costs = (Units) (Unit Costs) = (s)(Q)

then

$$dS/dt = (s) (dP/dt) + (P) (ds/dt) = (s) (P) (dP/dt) (1/P) + (P) (ds/dt) = (s) (P) (i) + (P) (ds/dt) = (S) (i) + (P) (ds/dt)$$

and

d(TC)/dt = (s) (dQ/dt) + (Q) (ds/dt) = (s) (Q) (dQ/dt) (1/Q) + (Q) (ds/dt)= (TC) (j)+ (Q) (ds/dt)

Noting that d(EBIT)/dt = dS/dt - d(TC)/dt with substitution,

then
$$d(EBIT)/dt = (i) (S) + (ds/dt) (P) - (j) (TC) - (ds/dt) (Q)$$

= (i) (S) - (j) (S - EBIT + (ds/dt) (P-Q)
= (i - j) (S) + (j) (EBIT) + (ds/dt) (EBIT/s)

Since (ds/dt)(1/s) = the real instantaneous percentage growth rate in sales or g_r ,

then d(EBIT)/dt = (i - j)(S) + (j + gr)(EBIT)

Therefore,

$$d(EBIT / S) / dt = \frac{[d(EBIT) / dt](S) - (dS / dt)(EBIT)}{S^2}$$

Simplifying, we get:

 $d(EBIT/S)/dt = (i - j) + (j + g_r) (EBIT/S) - [(dS/dt)/S] (EBIT/S)$

Since [(dS/dt)/S] = [(1 + i)(1 + gr) - 1] and (i) (gr) (EBIT/S) and (j-i) (EBIT/S) are both almost equal to zero,

then d(EBIT/S)/dt = (i - j)

APPENDIX B

The sustainable growth rate as given by Kisor [12] is:

 $g^* = (i - d) (ROE)$

where: g^* is the sustainable growth rate in earnings per share; d is the target dividend payout ratio; and ROE is the rate of return on equity.

Lerner and Carleton' s [13] sustainable growth rate model is:

 $g^* = b (1 - T) [r + (r - i) (L/E)]$

where: g^* is the sustainable growth rate of assets;

- *b* is the company's earnings retention rate;
- T is the company's tax rate;
- *r* is the rate of return on assets;
- *i* is the interest rate on borrowed funds;
- L is the company's total liabilities; and
- E is shareholders' equity.

In an enlightening article Higgins [7] derived the following sustainable growth rate model:

$$g^* = \frac{P(1 - d)(1 + L)}{t - P(1 - d)(1 + L)}$$

where: g^* is the sustainable growth rate in sales;

- *P* is the profit margin on new and existing sales after taxes;
- *d* is the target dividend payout ratio;
- L is the target total debt to equity ratio; and
- t is the ratio of total assets to net sales.

The sustainable growth rate concept was also researched by Ulrich and Arlow [18]. Their model is the following:

 $g^* = (NI/S) (S/TA) (1 - P) [1 + (D/E)]$

where: g* is the sustainable growth rate; *NI/S* is the profit margin on sales; *S/TA* is the total asset turnover; *1-P* is the retention ratio; and

D/E is the debt-to-equity-ratio.

In 1981 Johnson [9] published a paper qualifying Higgins' [7] model of the real sustainable growth rate. The real sustainable growth rate model restated by Johnson becomes:

$$g^{*}_{r} = \frac{(1+j) P (1 - d) (1 + L_{L}) - Cj}{f - (1 + j) P (1 - d) (1 + L_{L}) + C (1 + j)}$$

where: g_{r}^{*} is the real sustainable growth rate in sales;

- *j* is the uniform inflation rate;
- P is the profit margin on new and existing sales;
- *d* is the firm's target dividend payout ratio;
- L_L is the firm's target long-term debt-to-equity ratio;
- C is the ratio of nominal net working capital to nominal sales; and
- f is the ratio of nominal fixed assets to real sales.

In response to Johnson's paper, Higgins [8] demonstrated that a company's nominal sustainable growth rate equals its sustainable growth rate in the absence of inflation plus the inflation rate:

 $g^{*}_{rc} = P(1 - D)(1 + L) / T$

where: g_{rc}^* is the instantaneous real sustainable growth rate in sales;

- *P* is the profit margin on new and existing sales;
- *D* is the firm's target dividend payout ratio;
- L is the firm's target total debt to equity ratio; and
- T is the ratio of total assets to net sales.

Eisemann [6] revised Higgins' [7] model by allowing only spontaneous liabilities to grow at the same rate as sales without increasing other sources of debt. Eisemann's model is presented below:

 $g'^* = m (i-d)/[(TA/S) - L - m (1-d)]$

where: g'^* is the sustainable growth rate in sales;

- *m* is the profit margin;
- *d* is the dividend payout ratio;

TA/S is the ratio of total assets to sales;

L is the ratio of spontaneous liabilities to sales.

Lewellen and Kracaw [14] developed three models: the first, for the case of no inflation; the second, for the case in which there is inflation and interest rates follow Fisher's Effect; and the third, for the case in which there is inflation and interest rates follow Darby's Effect. These are, respectively, the following:

$$g = \frac{m(I - r)}{(I - B)(C + I + F - L)}$$

- where: g is the annual percentage instantaneous rate of sales growth;
 - m is the after-tax percentage profit margin on sales;
 - *r* is the firm's percentage dividend payout ratio of profits;
 - B is borrowed funds as a percentage of total book capitalization;
 - C is the ratio of cash plus accounts receivable to annual sales;
 - *I* is the ratio of inventory to annual sales;
 - F is the ratio of net fixed assets to annual sales; and
 - L is the ratio of spontaneous liabilities to sales.

$$g' = g + i \left[\frac{I+F}{(I-B)(C+I+F-L)} - \frac{B(I-T)}{(I-B)} - \frac{ZFT}{(I-B)(C+I+F-L)} \right]$$

where: g' is the instantaneous nominal sustainable growth rate;

- *i* is the inflation rate;
- Z is the annual percentage rate of depreciation; and
- T denotes the corporate income tax rate.

and lastly,

$$g' = g + i \left[\frac{I+F}{(I-B)(C+I+F-L)} - \frac{B}{(I-B)} - \frac{ZFT}{(I-B)(C+I+F-L)} \right]$$

APPENDIX C

Proof that the models of Kisor [12], Lerner and Carleton [13], Higgins [7] and Ulrich and Arlow [18] are all equivalent:

New Assets = New Financing
New Assets = New Debt + New Equity
New Assets = NI(1-P)(D/E)+NI(1-P)
New Assets = NI(1-P)(1+D/E)

$$(\Delta S)(TA/S) = NI(1-P)(1+D/E)$$

 $\frac{\Delta S}{S} = \frac{NI}{TA} (1-P) \left(1+\frac{D}{E}\right)$
 $g^* = \frac{NI}{S} \frac{S}{TA} (1-P) \left(1+\frac{D}{E}\right)$
 $g^* = (1-P) \frac{NI}{TA} \frac{TA}{E}$
 $g^* = (1-P) \frac{NI}{E}$
 $g^* = Retention Ratio × Return on Equity$

Kisor [12]

Ulrich and Arlow [18]

Starting with the model of Ulrich and Arlow [18]:

$$g^* = \frac{NI}{S} \frac{S}{TA} (I-P) \left(I + \frac{D}{E}\right)$$

$$g^* = (1-P) \left(1 + \frac{D}{E}\right) \frac{NI}{TA}$$

$$g^* = (1-P) \left(1 + \frac{D}{E}\right) \left[\frac{(EBIT - I)(1 - T)}{TA}\right]$$

$$g^* = (1-P) \left(1 - T\right) \left(1 + \frac{D}{E}\right) \left[\frac{(EBIT - I)}{TA} + \frac{D}{E}\left(\frac{EBIT - I}{TA}\right)\right]$$

$$g^* = (1-P) \left(1 - T\right) \left[\frac{EBIT}{TA} - \frac{I}{TA} + \frac{D}{E}\left(\frac{EBIT}{TA} - \frac{I}{TA}\right)\right]$$

$$g^* = (1-P) \left(1 - T\right) \left[\frac{EBIT}{TA} + \frac{D}{E}\left(\frac{EBIT}{TA} - \frac{I}{TA} - \frac{I}{TA}\right)\right]$$

$$g^* = (1-P) \left(1 - T\right) \left[\frac{FBIT}{TA} + \frac{D}{E}\left(\frac{EBIT}{TA} - \frac{I}{TA} - \frac{I}{TA} - \frac{E}{D}\right)\right]$$

$$g^* = (1-P) \left(1 - T\right) \left[r + \frac{D}{E}\left[r - i\left(\frac{D}{TA} + \frac{D}{TA}\right)\right]$$

$$g^* = (1-P) \left(1 - T\right) \left[r + \frac{D}{E}\left(r - i\right)\right]$$

Lerner and Carleton [13]

Starting with Higgins' model [7]:

$$g^{*} = \frac{P(I-d)(I+L)}{t-P(I-d)(I+L)}$$

$$g^{*} = \frac{(NI/S)(1-d)(1+D/E)}{(TA/S) - (NI/S)(1-d)(1+D/E)}$$

$$g^{*} = \frac{NI(1-d)(TA/E)}{TA - NI(1-d)(TA/E)}$$

$$g^{*} = \frac{(NI/E)(1-d)}{1 - (NI/E)(1-d)}$$

Starting with Higgins' model [7]: (CONT'D)

$$g^* = \frac{NI \ (1-d)}{E - NI(1-d)}$$

$$g^* = \frac{NI \ (1-d)}{Beginning \ of \ Year \ Stockholders' \ Equity}$$

 $g^* = Retention Ratio \times Return on Equity$

Kisor [12]