

EXCHANGE RISK: A CAPITAL ASSET PRICING MODEL FRAMEWORK

Conway L. Lackman*

INTRODUCTION

It should not be surprising that in times of international monetary instability, there is renewed interest in the definition and analysis of exchange risk. One analytical framework that seems particularly suited to the analysis of the problem of exchange risk is the well-known Capital Asset Pricing Model (CAPM). CAPM is a two parameter, single period model focusing on the expected return of an asset and the asset's riskiness. Risk is measured by the variance of the asset's rate of return over time measured by ex-post data. Generalizing this framework to handle assets denominated in different currencies (with an added element of risk due to the presence of foreign exchange and so the possibility of devaluation) appeared to be a straightforward extension of the CAPM in its domestic context. This view was advanced by Grubel who pointed out that the models of portfolio balance developed by Markowitz and Tobin explain the real world phenomenon of diversified asset holdings elegantly and properly. However, their analysis has not yet been applied explicitly to the explanation of long-term asset holdings that include claims denominated in foreign currency.[5]

The purpose of *this* paper is to present the development of the treatment of exchange risk in the CAPM framework in such a way as to be a useful introduction to the theory, as well as to provide some insight into the questions the theory must address and the problems it poses.

The assumptions and basic conclusions of CAPM are available from various books and articles written by the theory's codevelopers.¹ An abridged treatment is provided here for the reader's convenience. The assumptions underlying portfolio theory are:

1. r (the rate of return) adequately summarizes the outcome from the investment. Investors see the rates of return in a probability fashion.
2. Estimated risk attached to the investment is proportional to the variability of the return visualized.
3. The investment decision is based on only two parameters: $E(r)$ and σ , where $E(r)$ is the expected rate of return and σ is the standard deviation attached to the return. Since only two parameters are assumed, this is equivalent to the investor's utility function $U = f(E(r), \sigma)$.
4. $dU/dE(r) > 0$, $dU/d\sigma < 0$. [3]

These are the conditions necessary for investors to be Markowitz efficient.

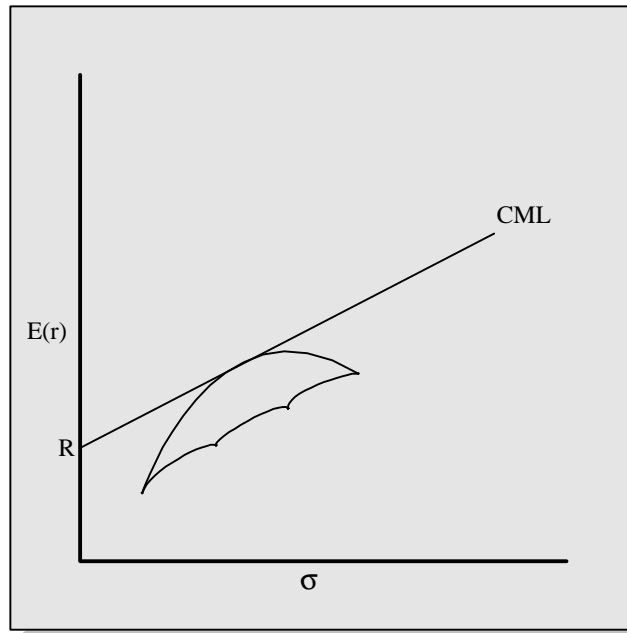
A theory of capital markets made up of Markowitz efficient investors requires some further assumptions.

1. All investors are Markowitz efficient and seek to attain the efficient frontier (defined below).
2. Any amount can be borrowed or lent at R (the riskless rate).
3. Homogeneous expectations, that is, all investors visualize identical probability distributions for expected rates of return.
4. All investors have a one period time horizon.
5. All investments are infinitely divisible.
6. No taxes or transactions costs.
7. Either no inflation and changes in interest rates, or all changes are fully known and anticipated.
8. The capital market is in equilibrium (no securities selling at other than equilibrium price). [3]

*Duquesne University

The assumptions listed above make possible the construction of a capital market line (CML) illustrated in Figure 1:

FIGURE 1



The efficient frontier is obtained by maximizing the rate of return available in the opportunity set of all portfolios (all combinations of securities) for a given level of risk. The capital market line (CML) is uniquely determined by the risk free rate of interest (R), and the tangency point with the efficient frontier. These assumptions allow the derivation of the separability theorem. The separability theorem implies that all investors should hold the same mix of stocks in their portfolios. They should then use borrowing or lending to attain their preferred risk class. That is, the investment decision is “separated” from the finance decision.

The shape of the efficient frontier depends on the covariance of the securities making up the opportunity set. This is seen most clearly in the two security case. The variability of return on portfolio p made of securities 1 and 2 is given by:

Equation 1

$$\text{Var}(r_p) = w_1^2 \text{Var}(r_1) + w_2^2 \text{Var}(r_2) + 2w_1w_2 \text{Cov}(r_1r_2)$$

The more negative the covariance term, the greater the potential for the reduction of risk through diversification, the extreme case being $\text{Cov}(r_1r_2) = -1$.^[3] w_1 and w_2 are the weights of securities 1 and 2 respectively in portfolio p. With this introduction, the discussion can turn to Grubel’s extension of the CAPM to an international context.

Grubel constructs a model of static equilibrium between two countries with independent monetary and fiscal policies initially in autarky. Population, wealth, and income are constant through time and he allows for three forms of holding assets: money, real assets, and bonds. The latter are issued by the government. The quantity of bonds in the market and the interest rate are regulated in such a manner as to maintain full employment.^[10] The last assumption allows Grubel to use a simple portfolio balance model. The domestic portfolios before trade or international contact are then constructed and the market assumed in equilibrium. The domestic portfolio in countries A and B respectively are characterized by:^[7]

$$E(R_A) = R_A$$

$$\text{Var}(R_A) = \sigma_A^2$$

$$E(R_B) = R_B$$

$$\text{Var}(R_B) = \sigma_B^2$$

The internationally diversified portfolio possible after trade is given by:[8]

Equation 2a

$$E(R_A R_B) = P_A R_A + P_B R_B$$

and the variance of the diversified portfolio:

Equation 2b

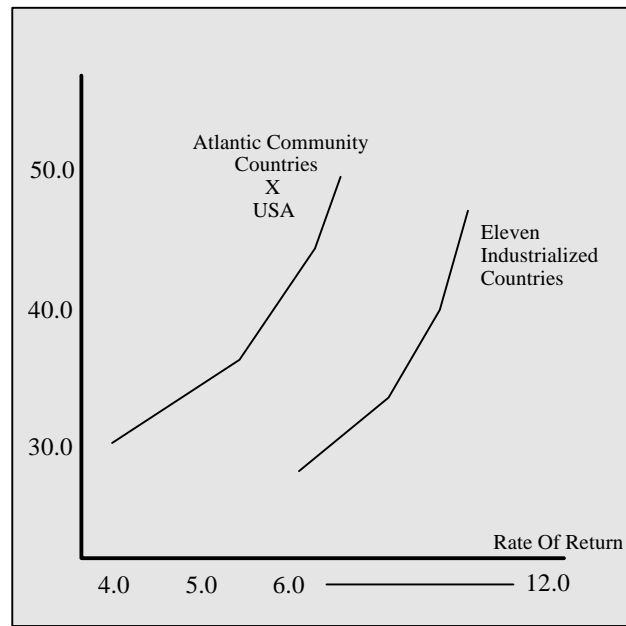
$$Var(R_{AB}) = P_A^2 \sigma_A^2 + P_B^2 \sigma_B^2 + 2P_A P_B \sigma_{AB}$$

where P_A and P_B are the proportions of securities A and B respectively in the average portfolio of country A.

Grubel uses this type of analysis to demonstrate the potential for risk reduction through international diversification. He identifies five determinants of the quantity of foreign assets demanded by residents of countries A and B: First, the size of total wealth assets held by the public. Second, the size of the interest rate differential given the variances and covariances of the two-asset returns. Third, the size of the risk differential given the earnings differential and covariance of returns...Fourth, the degree of correlation of returns on domestic and foreign assets. Fifth, the tastes of the public.[4]

Given the relationships established in the first part of this paper, Grubel's analysis can be seen to be a straightforward extension of the simple model. Grubel goes on to demonstrate possible gains from international diversification for Americans by computing rates of return and standard deviations from investing in 11 different countries as shown in Figure 2. The estimates are based on monthly observations on the indexes of common share prices (P), dividend yields on the shares in the index (Y) expressed as per cent per year, and the dollar exchange rate (X), defined as the price of one dollar.[9]

FIGURE 2



Rates of return converted to dollars, and the variances and covariances around a geometric mean were calculated for the common stock indices of the foreign countries studied and the United States. Sets of efficient internationally diversified portfolios were derived and are illustrated below for the 11 industrialized countries and a subset of the Atlantic Community Countries.

The analytical treatment of exchange risk is limited in this analysis, however, due to Grubel's assumption that: R_B , σ_B^2 , and $\sigma_{A,B}$ include an adjustment for exchange risk stemming from past variations in some shadow price of foreign exchange.[5] This assumption, while simplifying the analysis considerably, precludes an explicit treatment of exchange risk.

Solnik, similarly arguing for the benefits of international diversification, provides a method of treating exchange risk explicitly.[10] Solnik's objective was to determine how many securities are necessary to substantially reduce risk by diversifying internationally. His method involves randomly generating portfolios with increasing numbers of stocks. The method was repeated for domestic portfolios, international portfolios, and international portfolios unhedged against exchange risk. He also averaged the results on several portfolios of the same size to reduce sampling error. The study was based on weekly movements of stocks on the exchanges of the U.S. and seven European countries. The two countries results are shown in Figures 3A-3B.

FIGURE 3A

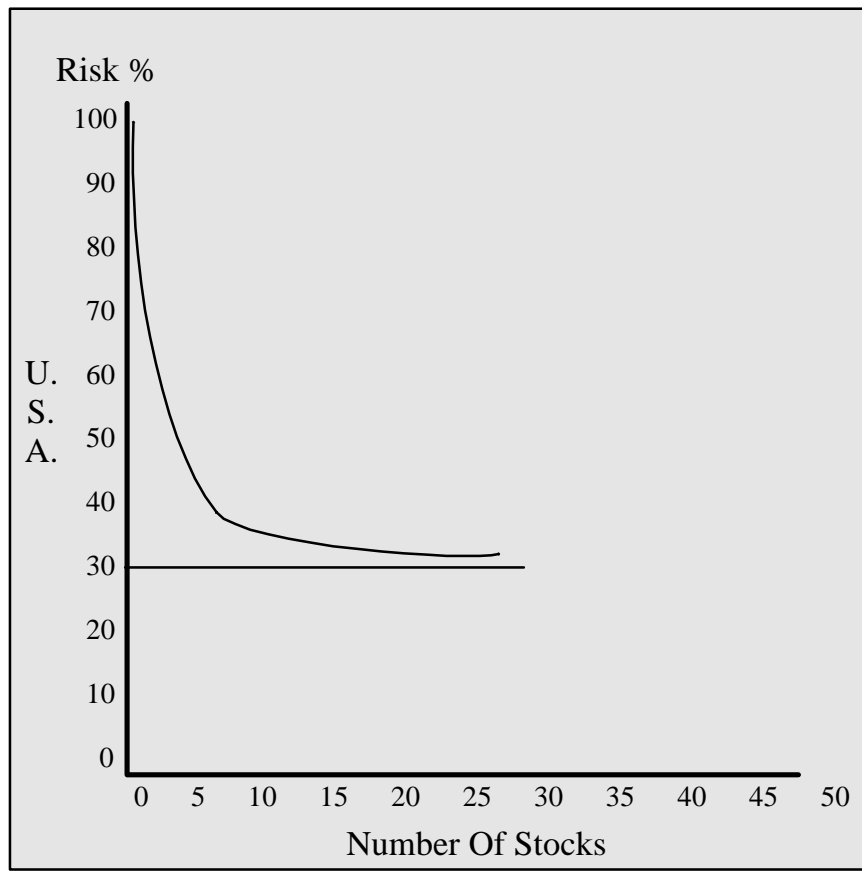
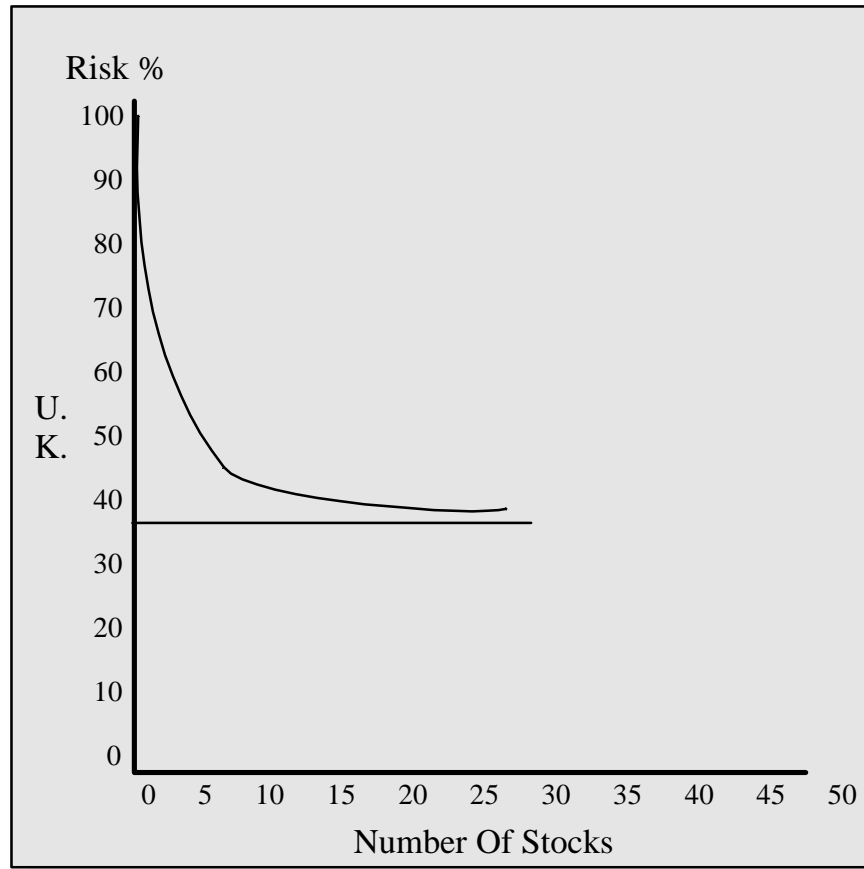


FIGURE 3B

In each case, the diminishing marginal reduction from adding an extra security to the portfolio is evident. However, in each country there is a certain level of risk which cannot be reduced through further diversification. This is what Sharpe termed “systematic risk”. Countries are seen to have different levels of systematic risk depending on the unique “economic, psychological, and political environment.”[3]

Solnik uses the same method to generate international portfolios. The portfolios are generated randomly (i.e. there is an equal chance of holding securities in each of the eight countries). For the sake of comparison, the U.S. and international portfolios are presented on Figure 4:[10]

The advantages of international diversification as depicted in the diagram below may be described as delaying the diminishing returns to diversification through providing investors with an expanded opportunity set. This is possible not only because of the increased number of stocks but also because of the relationship between the national markets. As noted above, much of the risk is accounted for by unique characteristics of the national markets. These studies are based on the positive but small correlation coefficients between price movements on the national markets.[7]

As noted earlier, Solnik’s analysis provides a way to incorporate exchange risk explicitly. The international portfolio was implicitly hedged against exchange risk in the above diagrammatical presentation; we can now observe the effects of exchange risk by comparing an uncovered portfolio in Figure 5.

FIGURE 4

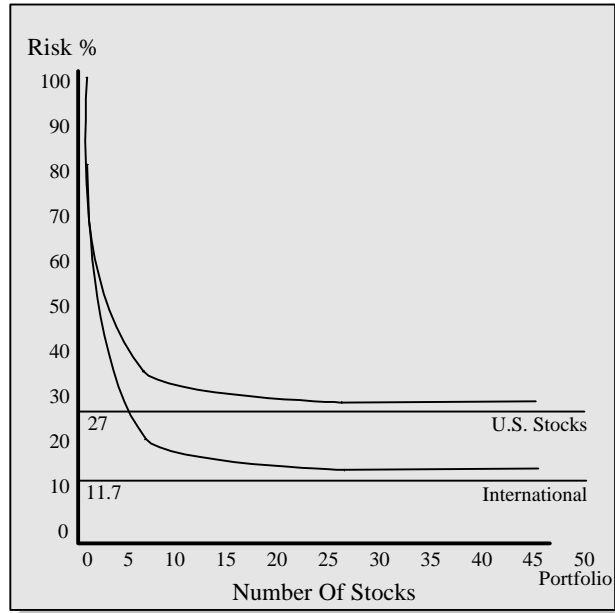
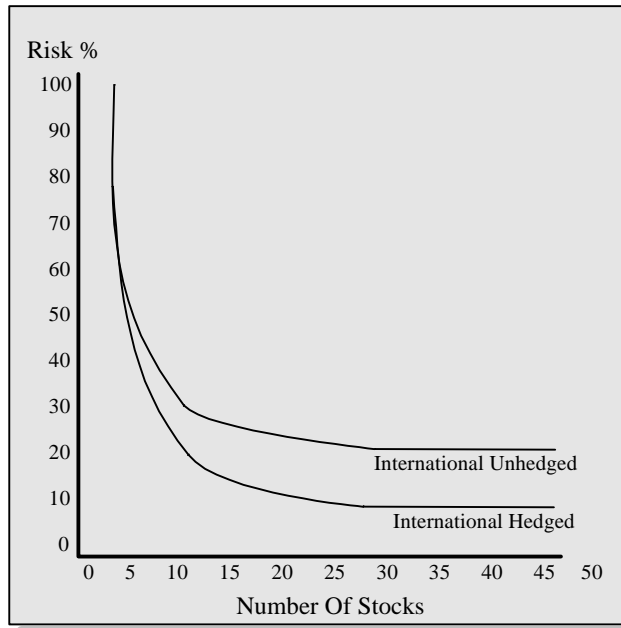


FIGURE 5



Solnik's analysis concludes with a recommendation of an uncovered international portfolio as a hedge against the devaluation of the dollar.

Levy and Sarnat raise some important question about the nature of exchange risk and the viability of the type of analysis pursued thus far. They argue that the existence of exchange risk makes it necessary to take into account the national identification of the investor. The possibility of changes in the external value of a country's currency unit (exchange rate) can, and usually does create differing sets of international investment opportunities for investment in different countries. The optimal investment proportions of an internationally diversified portfolio can no longer be expected to be the same in all countries.

To support this argument, mean rates of return and standard deviations of common stock indices for 26 countries were calculated and a quadratic programming technique used to find the locus of efficient portfolios. The objective function:

$$C = X'\Sigma X \text{ must be minimized subject to the restrictions that,}$$

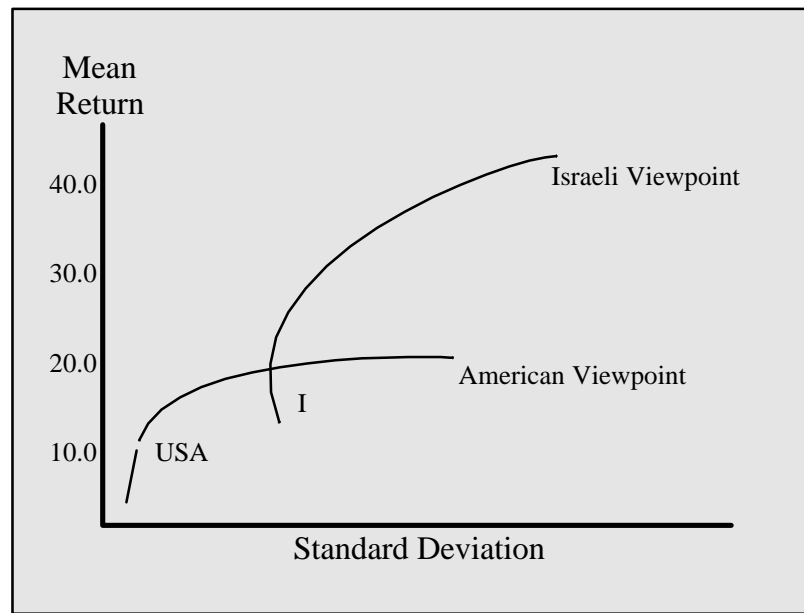
$$x_i \geq 0, X'R = E, \text{ and } X'I = 100.$$

where x_i is the proportion of the portfolio invested in country i , Σ is the variance-covariance matrix on the rates of return in countries i, j . R_i denotes the average rate of return on a given vector X . Finally $X'I = 100$ ensures that the portfolio proportions sum to 100%.

The efficiency frontier now calculated is used in conjunction with a concept of a CML discussed in the brief introduction to derive a one to one correspondence between the interest rate and the optimal portfolio for each given interest rate. This is used to show how the composition of the internationally optimal portfolio for a given investor changes with the interest rate.

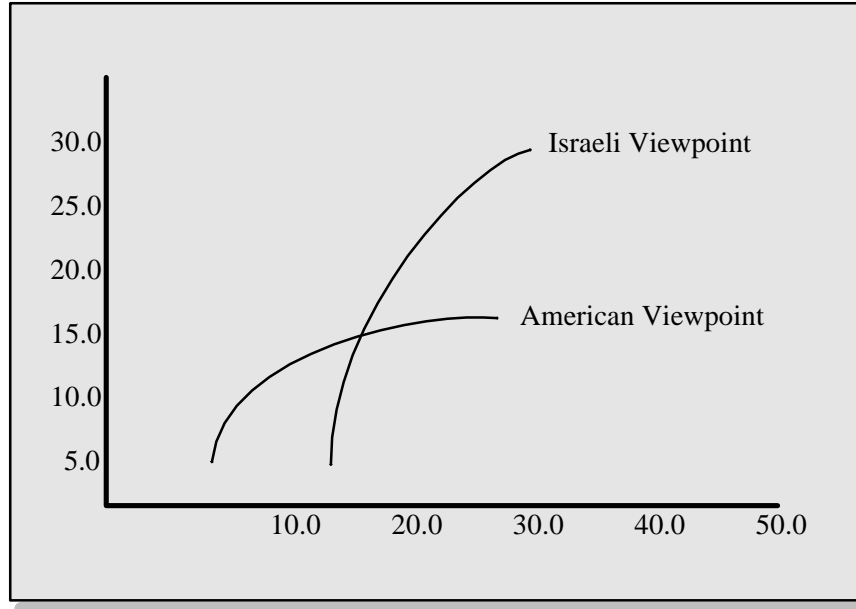
What immediately concerns the present discussion, however, is the treatment of exchange risk this approach makes possible. The authors compare efficiency curves for investors of two different nations to show the effect of exchange risk manifested in different efficient frontiers as illustrated in Figures 6A-B.

FIGURE 6A



The diagram above is in nominal terms. When calculated in real terms the effects of differential exchange risk are altered:

FIGURE 6B



The real efficient frontiers are calculated by deflating the index for the national rate of inflation. The problem of inflation is the most serious challenge to the usefulness of this type of analysis. The authors agree that the depreciation of Israeli currency was caused by that country's relatively high rate of inflation so that the large differential in the nominal returns on the same securities is illusory. Furthermore, in the long run, it could be argued that devaluation risk should not have an impact upon investment decisions.

Under fixed exchange rates the analysis would clearly be valid since adjustments could not take place instantaneously. For the case of floating exchanges the answer is not clear and would require further empirical investigation.

To try to answer this question several writers have extended the CAPM model into a dynamic long term framework. Since this departs from the basic model we set out to discuss, the results will not be considered in detail here. More importantly, there are serious doubts about the usefulness of such an extension. Both Solnik[8] and Grauer, Litzenberger and Stehle, in separate works have successfully generalized the CAPM into an international general equilibrium framework, but at great cost. Severely restrictive, unrealistic assumptions must be utilized to make the transition successfully. Among the more notable requirements are identical homothetic tastes among all consumers internationally, and identical expectations about the probabilistic nature of exchange rate changes. The method requires the maximization of a Von Neumann-Morgenstern utility function for lifetime income. The analysis requires an equilibrium relationship between interest rates and inflation, in addition to the assumption of a perfect international finance market.[4]

In this type of framework, Solnik finds that real exchange risk is due to different consumption preferences and fluctuating commodity prices.[9] It has been shown, however, that as long as international capital markets are not perfect, real exchange risk exists.[8]

Another argument could be made against the appropriateness of general equilibrium analysis especially in an international context where national monetary authorities domestic goals may be incompatible with an international general equilibrium and may indeed see its role as constraining the adjustment back toward an equilibrium.²

This brief invective against a general equilibrium analysis certainly does not eliminate the approach from consideration.³ It does make the case for the one period CAPM which, though less elegant and general in its assumptions and approach, is a potentially useful analytical technique.

ENDNOTES

1. See for example, W.F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* 19, 1964.
2. Black, Fischer, "International Capital Market Equilibrium with Investment Barriers," *Journal of Financial Economics*, 1974, pp. 337-352. See especially pp. 348-350. For a discussion of the effects of segmented markets (e.g. limited information) and its impact on exchange risk see Adler, Michael and Dumas, Bernard, "Optimal International Acquisitions," *Journal of Finance*, March 1975, pp. 1-19, especially pp. 10-19.
3. See for example, Kaldor, Nicholas, "Conflicts in National Economic Objectives," *Economic Journal*, March 1971, pp. 1-16.

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