Capital Budgeting When Projects Have Unequal Lives and Costs of Capital

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Abstract

This paper addresses some issues in capital budgeting that have not been completely resolved in the literature, and suggests a more comprehensive and realistic methodology for project evaluation and selection in the situation complicated by differing project life and costs of capital.

I. Introduction

The standard textbook coverage of capital budgeting has changed very little over the past twenty years, especially in the treatment of special topics such as investment projects with unequal lives. This is notable because, in general, financial economists have found promising new approaches in the search for understanding financial markets, the behavior of participants in these markets and other areas of concern to both practitioners and academics. The fact remains, however, that the purchase of real assets is one of the most commonly encountered and important decisions that firms make.

The organization of this paper is to first review and comment on the work of Emery (1982) to provide a framework for the analysis of projects when investment alternatives have unequal lives. The drawbacks and deficiencies of traditional capital budgeting techniques in dealing with projects of differing length of life and cost of capital are discussed. An alternative to the traditional unequal life methodology that allows for a more unified and realistic treatment of these complications is suggested.

II. General Framework for Capital Budgeting when Projects have Unequal Lives

In general, capital budgeting when projects have equal lives and equal cost of capital is a well-understood, relatively simple process. Standard texts agree that the organization confronted with an investment decision should use discounted cash flow techniques to compare the current value of the expected benefits of an activity to the costs. In general, the benefits of the investment opportunity are compared to the costs through the estimation of net cash inflows. These net inflows are then discounted to the present using the appropriate cost of capital. This approach may break down slightly when there are mutually exclusive alternatives with unequal lives.

Commonly, texts suggest that, when faced with the choice among alternatives with different maturities, the benefits and costs should be adjusted to some common life before a choice is made. [See, for example, Brealey and Myers (2003), Brigham and Ehrhardt (2002), Grinblatt and Titman (2002); Ross, Westerfield and Jaffe (2005), Smart, Megginson and Gitman (2004), and Van Horne and Wachowicz (2001).] One way to do this is with the replacement chain method. The investor must assume that the alternatives can be repeated until each provides cash flows for the same length of time. Under this method, the one-cycle NPV of each alternative is calculated. Then, assuming that this NPV will repeat in future cycles, the present value of the entire NPV stream is calculated. For convenience, we call the present value of the NPV stream the "total NPV." The total NPV of the now comparable cash flows can be used to make the choice.

Another method is the equivalent annuity (EA) method in which the actual cash flows of each alternative are transformed into an annuity before being extended to cover the common length of life. It is sometimes suggested in texts that an appropriate common length of life is infinity. This leads to the conclusion that the alternative with the largest annuity payment should be selected. Although it is certainly within the realm of possibility that certain projects might be considered infinitely lived, this adjustment may have originally been developed as a way to simplify the math by using the present value of a perpetuity formula.

Emery (1982) compares the expected length of the activity that requires capital investment, which he calls the project, to the useful economic life of the alternatives that facilitate the project. The first possibility (Case One) is that the economic life of the alternative will determine the life of the project. His example is the extraction of a fixed amount of natural resources. The alternatives may complete the process over different time periods. Emery suggests that since there can be no replication, no adjustment to the alternatives' lives should be made.

A second possibility (Case Two) is that the project life may be shorter than the economic life of the alternatives. For example, a firm might decide to produce a certain good for a fixed amount of time. The machines used in the production process may have an economic life that is longer than the period in which the firm wants to produce the good. Emery's example is the construction of a building. The alternatives should be evaluated over the production period, with salvage of the machines providing a cash flow in the final period.

Emery's Case Three involves investment alternatives in which the project's life is longer than the lives of the alternatives. Equipment or machinery must be replaced prior to the completion of the project. According to Emery the evaluation period should be the lesser of the project life or the lowest common multiple of the alternatives' lives. The final possibility foreseen by Emery (Case Four) is that the project itself has a length of life that falls between that of the alternatives. One alternative must be replaced before the project is finished, but another will last longer than the project. In this case, Emery suggests that the solution is to evaluate the alternatives for the life of the project. In Case One the project's life is defined by the alternatives. In Cases Two, Three and Four the project's life is fixed regardless of the alternative used.

In general, standard textbook coverage does not take advantage of the insights provided by Emery. Discussion of unequal lives is typically limited to an example that may fall into Case Three or Case Four, and the lowest common multiple of the alternative lives is chosen as the evaluation period. No attempt is made to describe whether the alternatives are designed to complete the same project or are unique projects that may be mutually exclusive for some other reason. Rarely is it specified whether the projects can actually be repeated. One important shortcoming of these tendencies is that the assumption of cash flow repetition is implicitly used without consideration for the potential impact on firm value of equipment salvage. This occurs even though these same texts discuss salvage as an important component of project cash flow, albeit typically in a different section or chapter of the book. No attempt is made to reconcile these two capital budgeting topics. In addition, texts typically assume the same cost of capital for all the alternatives and all the various components of net cash inflows. Beedles and Joy (1997) and Musumeci (1999) allow different costs of capital for the alternatives' net cash flow and point out some problems associated with the use of EA and replacement chain within the framework of Emery's Case Three.

III. The Impact of the Cost of Capital on Unequal Life Techniques

When mutually exclusive investment projects have unequal lives and the same cost of capital, the replacement chain and equivalent annuity (EA) always yield the same selection. However, if financing costs differ, either a conflicting selection may result as in Beedles and Joy (1997), or both methods may select the wrong alternative as described by Musumeci (1999).

Beedles and Joy (1997) illustrate that when costs of capital are not the same, one cannot simply compare the EA of competing alternatives. They also show that the EA method may lead to an incorrect decision. Pilotte (2000) points out that, although not explicit in the methodology, the ultimate comparison by the EA method is still the total NPV of each alternative; that is, the present value of the annuity stream. The only time it is justifiable to choose from among alternatives using only the EA, as suggested by most standard textbooks, is when the projects have the same cost of capital. The reason is that with equal cost of capital, the alternative with the highest EA will automatically have the highest total NPV regardless of the length of life used. Therefore, extending EA analysis to include the calculation of the total NPV based on a common length of life is identical to that used in the replacement chain method. The two methods would choose the same alternative. Henceforth, this paper will call this extension the modified EA method (MEA).

Musumeci (1999) extends the analysis of Beedles and Joy (1997) to show that the selection among competing alternatives depends on the common length of life chosen for both the replacement chain and the MEA methods. For a particular length of life, the two methods will select the same alternative. But if a different length of life is used, the methods may choose a different alternative. He suggests, like Emery, choosing a common length of life as close to the life span of the investment opportunity as possible. Pilotte (2000) reaches the same conclusion.

IV. MNPV and Emery Case One

Emery (1982) suggests that in Case One, where the project life is determined by the alternative used and the alternatives cannot be repeated, one should simply select the alternative with the highest one-cycle NPV. This method is not always correct. Consider coal mining. Let Alternative A completely recover the coal over 10 years and Alternative B in 8 years. Assuming that A and B have the same NPV, Alternative B may be superior if the cash inflows from B can be reinvested at the beginning of year 9 into another positive-NPV project. In fact, the NPV assumes that future cash flows during the alternative's life cycle will be reinvested to the end of the cycle and earn a rate of return equal to the alternative's cost of capital; in other words, it assumes that cash flows will be reinvested in a zero-NPV project. This assumption is not true for companies that still have positive-NPV investment opportunities. Cash inflows will be reinvested over time in the firm's future investment opportunities in general; therefore, it may be more reasonable to assume the same reinvestment rate for all the alternatives, and this rate should be equal to the firm's average rate of return of investments not the cost of capital. [See Meyer (1979).] An approach that explicitly considers the reinvestment assumption is the Modified Net Present Value (MNPV) as defined by equation (1).

$$MNPV = -I_0 + \frac{\sum_{t=1}^{n} CF_t (1+RR^*)^{n-t}}{(1+K)^n} \tag{1}$$

I_0 is the initial investment, CF_t is the net cash inflow for period t, RR* is the firm's average return on investment, and K is the alternative's cost of capital. The lower case n is the length of life of the alternative. The major flaw of this approach is that multiple discount rates are necessary if the reinvestment opportunities have a different level of risk than the alternative's direct cash flows or if the firm expects to have positive-NPV projects in the future. By forcing a single discount rate, MNPV implies that cash flows can only be reinvested in an asset with similar risk.

The MNPV rule is modified, therefore, to select an alternative in Case One as shown in (2).

$$MNPV^* = -I_0 + \sum_{t=1}^{N} \frac{CF_t (1+RR^*)^{N-t}}{(1+K)^t (1+K_c)^{N-t}} \tag{2}$$

I_0 is the initial investment, CF_t is the net cash flow for period t, RR* is the firm's average return on investment, K is the alternative's cost of capital, and K_c is the company's cost of capital. N is the project length of life. [Note the distinction between the n of formula (1) and the N of formula (2).] The reinvestment rate RR^* can be estimated based on the rate of return of currently held assets, taking into account future technological innovation, inflation, competition, or other estimable factors. If assets as a whole in the future are considered a positive-NPV venture (not a strong assumption for a going concern), RR^* is greater than K_c. Since the reinvestment will be in the firm's investment opportunities in general, the corresponding discount rate during each cash flow's reinvestment period should be the company cost of capital (K_c). The decision rule is then to select the alternative with the highest $MNPV^*$.

Using $MNPV^*$, the alternatives' lives can be made equal by allowing for the reinvestment of the released resources; for example, the 8-year alternative's $MNPV^*$ includes an estimate of what can be earned in years nine and ten from the reinvestment of the alternative's cash flows. Even though the new net cash flows for the last two years are zero, the firm is still earning reinvestment income on prior year's cash flows. These earnings should be considered when choosing between the two alternatives. $MNPV^*$ can be calculated in either nominal or real terms as long as consistency is maintained; that is, if the cash flows are nominal (real), the cost of capital should also be nominal (real).

Emery's (1982) suggestion that one can just compare the one-cycle NPV of the alternatives to make a selection in this case is correct only if the firm expects to have no positive-NPV reinvestment opportunities. Thus, MNPV* allows analysts to evaluate alternatives for the same time period while considering different levels of alternative and reinvestment risk and the

possibility that the reinvestment opportunities are economically beneficial. MNPV* is an improvement over the traditional approach.
For example, consider the extraction of natural resources as suggested by Emery. Alternative A, which will extract the entire amount over an 8 year period, has an initial investment of $5,000 and net cash flow of $1,150 in each year. The appropriate cost of capital is 10%. For the same initial investment, Alternative B has lower operating cost allowing the generation of $1,400 per year for six years at the same cost of capital. The company's overall cost of capital is 11% and it expects to have reinvestment opportunities in the future that earn an average return of 12%. The traditional NPV for Alternative A is $1,135.165. That for B is $1097.365. Using MNPV* reveals that Alternative A is really worth $1,360.304 when reinvestment is allowed in average projects. However, the MNPV* of Project B is $1,364.975. Because it does not consider the benefit in years 7 and 8 from the early release of resources provided by Alternative B, the use of NPV would result in the less optimal selection of Alternative A. This occurs even though the alternatives' costs of capital are identical. Lowering the cost of capital for B to 9.75% increases B's MNPV* to $1,411.50, making the difference between the alternatives more pronounced.

V. Improving the Other Cases

In Case Two, in which all the alternatives have a length of life longer than the project, the alternatives would not be repeated and should be truncated to match the project life. Therefore the common length of life is equal to the project life. Include the salvage of the assets as a cash inflow. Again the alternative with the highest MNPV* should be selected.

For example, let Alternative A have an economic life of 6 years and a cost of capital equal to 13%. The initial investment is $4,000 and the subsequent cash flows are $1,200 per year. The equipment can be sold for $120 in year five. Alternative B has an economic life of 7 years, will cost $4,400 and generates cash flow of $1,300 per year. It can be sold in year five for $200. The cost of capital for B is also 13%. The project will only be viable for five years. The company has an average return of 12% and a cost of capital of 11%. NPV suggests that the firm should buy Alternative A, because the net present value of $285.81 is greater than B's NPV of $280.95. However MNPV* suggests the firm should buy B. Due to the larger inflows, the MNPV* of B is $374.228 and that of A is only $371.909.

In Case Three, all the alternatives have a length of life shorter than the project life. Emery (1982) suggests that the alternatives should be repeated to a lowest common length of life or equal to the project life. That is, if the common length of life for the alternatives is longer than the project life, the alternatives will be truncated in their last cycle to match the project life. If the alternatives have the same cost of capital, conventional wisdom would suggest using either the replacement chain or EA method to make a selection. Since these approaches are based on the conventional NPV, they inherit the reinvestment rate problem described above.

The value of an alternative should include the return from released resources that can be reinvested in other activities. This aspect of MNPV* makes it a more accurate measure of value than NPV, even when the alternatives' length of life is the same. The disadvantages of NPV-based methods are magnified if the alternatives do not have the same cost of capital.

Musumeci's (1999) argument suggests using a common length of life for the alternatives as close to the project life as possible, since a different length of life may result in a different selection by either the replacement chain or the MEA method. However, there is an additional problem. Both the replacement chain and the EA (or MEA) methods assume that an alternative can be repeated with the same cash flow stream and cost of capital in each cycle. Is it realistic to assume an unchanging cash flow stream? The level of future cash flows may be influenced by technological innovations, inflation, and/or changes in the level of competition. Since it is difficult to predict the joint impact of these factors on future cash flows, assuming the same cash flow stream for every cycle may be an acceptable simplification. The cost of capital, in the context of CAPM, is a function of the basic interest rate, inflation, the risk of the project and the market risk premium. Again, it is difficult to predict the joint effect of these variables on the future cost of capital and assuming the same cost of capital for each cycle may be acceptable. But when both the same cash flow stream and cost of capital are repeated in every cycle, the result may have disturbing implications.

Both the replacement chain and the EA (MEA) methods repeat the first-cycle NPV, which is usually positive, to the common length of life. This approach implies that the project can consistently earn an abnormal return equal to that of the first cycle. This is an unlikely simplification; the project is typically expected to experience diminished returns due to competition. The replacement chain and the EA (MEA) calculations, therefore, may misestimate the alternative's value.

Instead, one should use just one cycle for each alternative and the length of the cycle is equal to the project life. A capital investment such as machine replacement would not be treated as the beginning of another NPV cycle but merely a cash outflow. The $MNPV^*$ of an alternative is calculated based on estimated cash flows over the life of the project and a cost of capital appropriate to the alternative's risks and life span. Cash flows at each time should be forecasted based on factors relevant to the time. In this fashion, technological innovations, inflation, and/or competition can be taken into account over the entire length of the project. The decision is to select the alternative with the highest $MNPV^*$. This approach, in effect, converts Case Three into Case Two.

Consider the example of a project that is expected to have a useful life of 12 years. We can implement the project with either of two alternatives. Alternative A has a three year cycle that involves spending $3,100 now to earn inflows of $1,200 per year. Alternative B has a four year cycle, an initial investment of $3,400 and is expected to earn $1,300 per year. Using the EA approach, the one-cycle NPV for A is -$115.778 resulting in a total NPV of -$317.218 if the cost of capital is 10%. Project B has a one-cycle value of $720.825 and a total NPV of $1,549.43 at the same capital cost. Traditional treatment of the investment decision suggests that B is the better choice. In fact, A is not considered a value-enhancing choice.

Using MNPV* and assuming that the analyst can forecast cash flows for each cycle, a different result could be obtained. Let the average return to the company be 12% and the overall cost of capital equal 11%. The table below shows the cash flow estimates for each alternative. In general, these alternatives experience decreasing costs and inflows each cycle, perhaps due to improvements in technology and new competitors. The NPV of A is found to be -$23.868. That of B is -$444.375. However, the MNPV* of A, $182.953, indicates that it is the better choice. The MNPV* of B is -$162.66.

Cash Flows for Alternatives in Case Three

Year	Alternative A	Alternative B
0	-3,100	-3,400
1	1,200	1,300
2	1,200	1,300
3	-1,500	1,300
4	1,100	-1,800
5	1,100	1,000
6	-1,300	1,000
7	1,000	1,000
8	1,000	-1,600
9	-1,200	960
10	900	960
11	900	960
12	900	960

As in Case One, the numerical examples for Cases Three and Four are calculated using the same costs of capital for each alternative. Differences in selection based on NPV and MNPV* occur even without differing costs of capital. The MNPV* method explicitly allows the analyst to account for variations in capital costs, as well as any other potential difference among alternatives without suffering from the shortcomings of the replacement chain, EA and NPV.

If the project life falls between the competing alternatives' lives (Case Four), the longer alternative's life should be truncated to match the project life, similar to the treatment in Case Two and Three. The shorter alternatives should be treated in the same fashion as those in Case Three. This results in all the alternatives having the same length of life as the project. Again, for the reasons cited above, the decision should be based on the alternatives' $MNPV^*$.

VI. Conclusions

One of the potential strengths of Finance as a social science is the ability to apply a small number of theoretical insights to a wide range of questions faced by both practitioners and financial economists. This strength is not used to advantage in the traditional treatment of the topic of capital budgeting with unequal life investment alternatives. Finance texts do not offer a clear, consistent approach to this problem. In addition, the most frequently cited approaches only hold under a very narrow and often unrealistic set of assumptions. This lack of clarity is complicated when alternatives have differing lives. The use of a modified NPV method is suggested as an alternative that allows for a more unified and realistic treatment of these complications because it requires no assumptions about project replication, it can easily handle alternatives of different risks and it allows the analyst the opportunity to consider salvage value reinvestment income, and discount rates in a consistent manner.

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