Financial Decisions, Winter 2013, Article 4

A Note on Long Real Interest Rates and the Real Term Structure

Joseph C. Smolira[*,1] and Denver H. Travis[**,2]
[*]Belmont University
[**]Eastern Kentucky University

Abstract

Orthodox term structure theory holds that real interest rates are constant across all maturities. With the introduction of inflation indexed securities by various governments, the real interest rate is directly observable. The yields of these inflation linked securities show that the real rate changes and exhibits a term structure. We use using monthly U.S. Treasury Inflation Protected Securities (TIPS) over the period 1999 – 2009 to estimate the infinite maturity real interest rate. Based on the Cox, Ingersoll, and Ross (1985) term structure model and a constant-drift adaptation of that model, we find that implied long maturity real zero-coupon rates did fall substantially during the last half of this period.

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[1]Contact author

[2]Part of this work was performed while the author was at the College of Business Administration of the California State University, Sacramento and was partially supported by a research grant at this institution.

1. Introduction

According to Fisher (1930), the real rate of interest is constant. Since historically no observable measure of the real interest rate was available, it was generally assumed that the real interest rate was flat and exhibited a constant term structure. With the introduction of "index linked" gilts (ILGs) in the U.K. in the early 1980s and Treasury Inflation Protected Securities (TIPS) in the U.S., among others, the real rate of interest is now directly observable.

Even before the introduction of securities that offered a guaranteed real rate of return, empirical research such as Nelson and Schwert (1997), Walsh (1987) and Rose (1988) found that real interest rates vary and exhibit a term structure. Using ILG yields, Brown and Schaefer (1994) and Evans (1998) estimated the pretax real term structure, and Aziz and Prisman (2000) extended this research to estimate the aftertax real term structure. Using TIPS, Beechey and Wright (2009) examines economic factors that affect the real interest rate. While these and other researchers have examined real interest rates, one limitation is that their results are limited to a 10- to 20-year horizon because of the availability of directly observable data.

While some researchers have used the Fisher effect to estimate real interest rates, for example, Lim and Ji (2011), and Choi and Devereaux (2006), research such as Evans (1998) indicates that the Fisher effect may not hold. Other researchers have used alternate methods to estimate the real interest rate. For example, Buraschi and Jiltsov (2005) examine the expectations hypotheses by developing a general equilibrium model to explain deviations. Their results indicate that that at medium- and long-term horizons, the Fisher effect is unable to explain real interest rates. Rapach and Weber (2004) use the Ng and Pearson unit root test to find that the international real interest rates are nonstationary. In this paper, we use the yields on TIPS to directly measure the real interest rate.

Term structure theory for nominal interest rates holds that long-term, or infinite maturity nominal interest rates, cannot fall. Specifically, Dybvig, Ingersoll, and Ross's (DIR, 1996) argument is that if long (i.e., infinite maturity) forward and zero-coupon rates can decline, then, asymptotic arbitrage opportunities exist. However, Jordan, Jordan, Smolira, and Travis (2008) show that for the 1990-2000 period, long-term nominal interest rates did fall substantially.

Our primary goal in this paper is to empirically examine the behavior of long-term real interest rates. Concurrently, we examine the behavior of long-term nominal interest rates. To estimate the real interest rate, we extrapolate the infinite maturity real rate using U.S. Treasury TIPS. A fundamental problem for such research is that rates for maturities much longer than 20 years are unobservable. We estimate a commonly-used term structure model using monthly data on U.S. Treasury zero-coupon TIPS and nominal zero coupon bonds covering the period January 1999 to July 2009 and examine the real and nominal long maturity zero-coupon rates implied by the fitted term structures. The decline in interest rates during this period coincided with the latter part of "The Great Moderation" when there was a "substantial decline in macroeconomic volatility" (Bernanke, 2004). To our knowledge, we are the first to extensively investigate the empirical behavior of infinite maturity real rates. Our primary finding is that long real interest rates are relatively stable over the first half of period; however, these rates fall substantially over the last half of the decade.

The remainder of the paper is organized as follows. The next section describes our data and methods. Section III presents the empirical results and Section IV concludes the paper.

2. Data and Empirical Methods

Daily observations of estimated zero coupon interest rates for both TIPS and nominal Treasuries were obtained from the U.S. Federal Reserve Economic Research & Data website for the period of January 4, 1999 to July 27, 2009. The TIPS zero coupon data contains yield observations at each maturity from 5 to 20 years for January 4, 1999 to December 31, 2003. From January 2, 2004 to July 27, 2009, the dataset contains yields at each maturity from 2 years to 20 years. The nominal zero coupon dataset contains observations at maturities of one year to 30 years. For comparison purposes, the maturity range of 2 years to 20 years was selected for term structure analysis on both the TIPS data and the nominal Treasury data when data was available. The maturity range of 5 years to 20 years was used for the TIPS data from 1999 to 2003 due to data availability.

Jordan, Jordan, Smolira, and Travis (2008) develop a method to use available term structure data to extrapolate out to the very long maturity interest rates that are the subject of DIR (1996). This method involves first fitting the cross section of a daily observation of interest rates to the term structure model yield equation as prescribed by Brown and Dybvig (1986). From the estimated parameters, the yield at a very long or infinite maturity can be calculated. This methodology is highly tractable for most single factor term structure models.[1]

The basic model for this study is the Cox, Ingersoll, and Ross (1985) term structure model, as shown in Equation 1.

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz \qquad (1)$$

where *dr* is the change in the short interest rate *r* over a time interval *dt*, κ is the speed of adjustment coefficient, θ is the stationary point, and σ is the volatility term. In addition, a constant drift adaptation of the CIR model is used for comparison, shown by Equation 2.

$$dr = \mu dt + \sigma\sqrt{r}dz \qquad (2)$$

[1] Although the term structure literature includes various multifactor term structure models, Jordan, Jordan, Smolira and Travis (2008) document that over 97% of the variation of U.S. Treasury STRIPS is accounted for by one factor.

where μ is the constant expected drift in the short interest rate. As seen in the results section below, the constant drift model often converges more easily than the CIR model, especially when the term structure is very flat.

In a manner similar to Brown and Dybvig (1996), the two term structure models were cross-sectionally fit to the 2nd Wednesday of each month in order to create a monthly data series that attempts to avoid end-of-the-week effects and end-of-the-month effects. The Marquardt method, which is a variant of the Gauss-Newton method, was used to perform the nonlinear regressions on each cross-section. When the term structure models would not converge, the adjacent Tuesday or Thursday was used. If neither of those days would allow for convergence, the following Wednesday or adjacent Tuesday or Thursday was used. For a number of months, none of these daily term structures would converge with the model. For comparison at each observation, the same day of the month was used for both the TIPS data and the nominal Treasury data and for both term structure models. Once the parameters of the term structure models were estimated, the infinite maturity yield was then calculated. The results of this analysis are displayed in the next section.

3. Empirical Results

Figure 1 displays the results of the estimated infinite maturity, or asymptotic, zero coupon interest rates for both TIPS and nominal Treasuries when using the CIR (1985) model. Vacant areas are the result of a lack of model convergence for that particular monthly data point. As shown, both the TIPS and nominal asymptotic rates did fall substantially over the period from 1999 to 2009. The TIPS asymptotic rate fell from over four percent to less than two percent. The nominal asymptotic rate fell from over seven percent to less than four percent. The yield spread also declined substantially over the period, falling from over three percent to less than two percent.

Figure 1.
Estimated Asymptotic Zero Coupon Rates Implied by the CIR Model

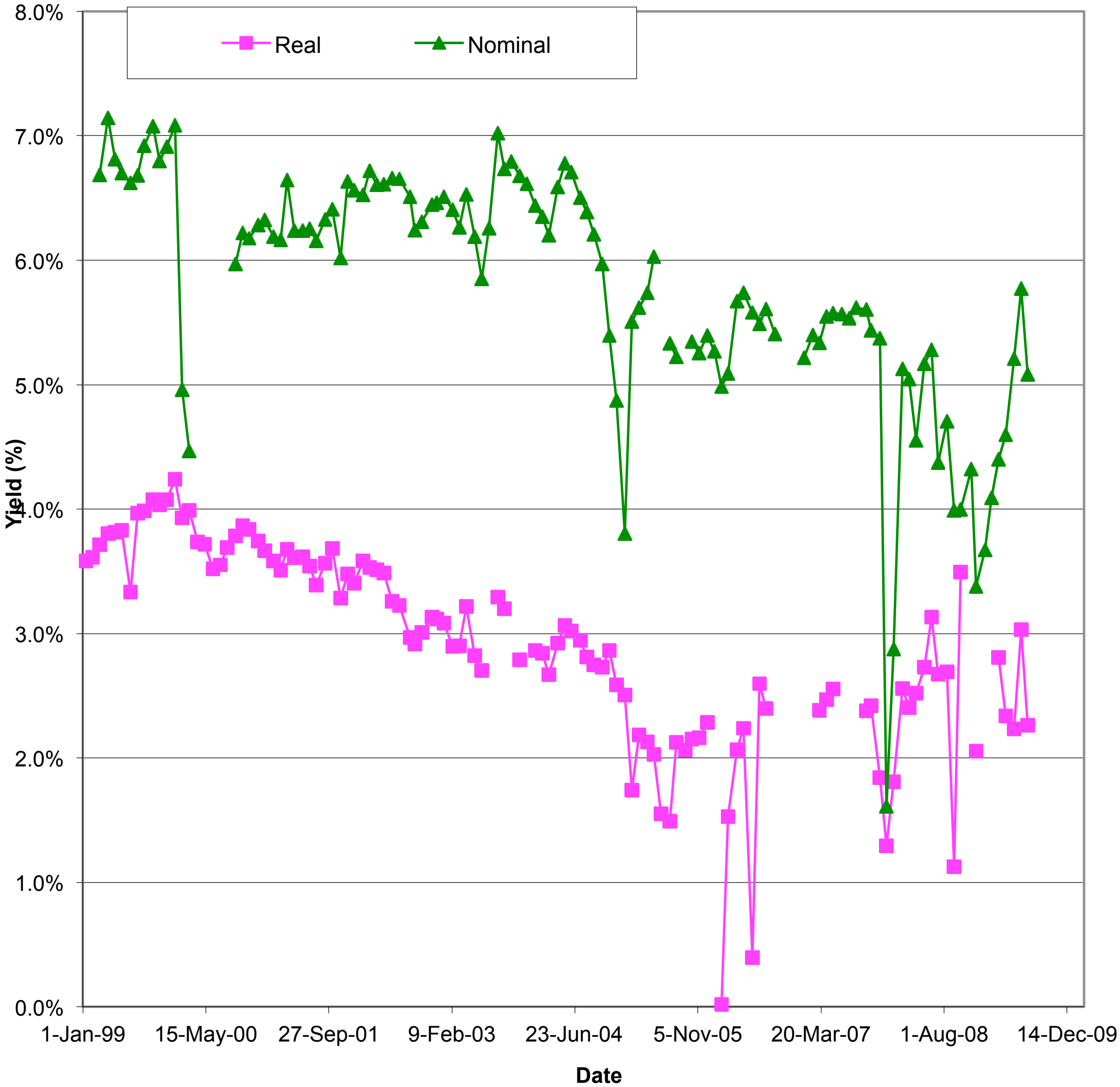

Figure 2.
Estimated Asymptotic Zero Coupon Rates Implied by the Adapted CIR Model

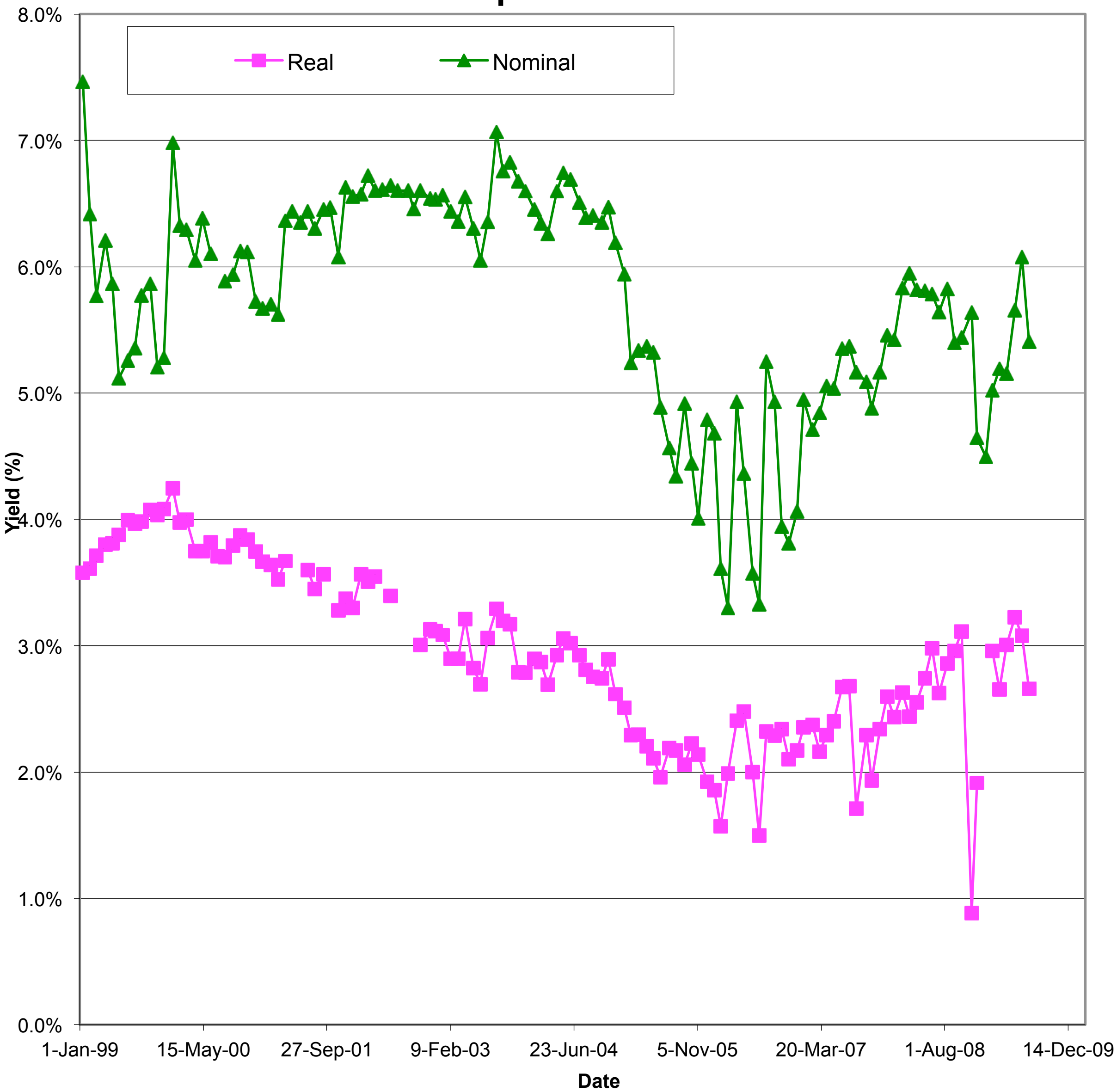

Figure 2 presents the results when using the constant drift adaptation of the CIR model. These results closely resemble the CIR results from Figure 1. Model convergence was more frequent when using the constant drift model. As noted by Jordan, Jordan, Smolira and Travis (2008), it appears that the CIR model may be over-parameterized in flatter term structure scenarios.

Apparently the estimated infinite maturity zero coupon interest rate does fall, both in nominal and real (TIPS) terms, for the period of early 1999 to mid 2009. Along with the 1990 to 2000 evidence from Jordan, Jordan, Smolira, and Travis (2008), these results run counter to the predictions of Dybvig, Ingersoll, and Ross (1996). It should be noted that Dybvig, Ingersoll, and Ross (1996) made their prediction for a world where there are no market frictions. If there are market frictions that cause the asymptotic rate to fall over time, one could assume that a weaker prediction is that the asymptotic rate should at least vary less than shorter term rates.

Figure 3.
Real Interest Rate & Nominal Interest Rate:
Averages & Standard Deviations At Different Maturities

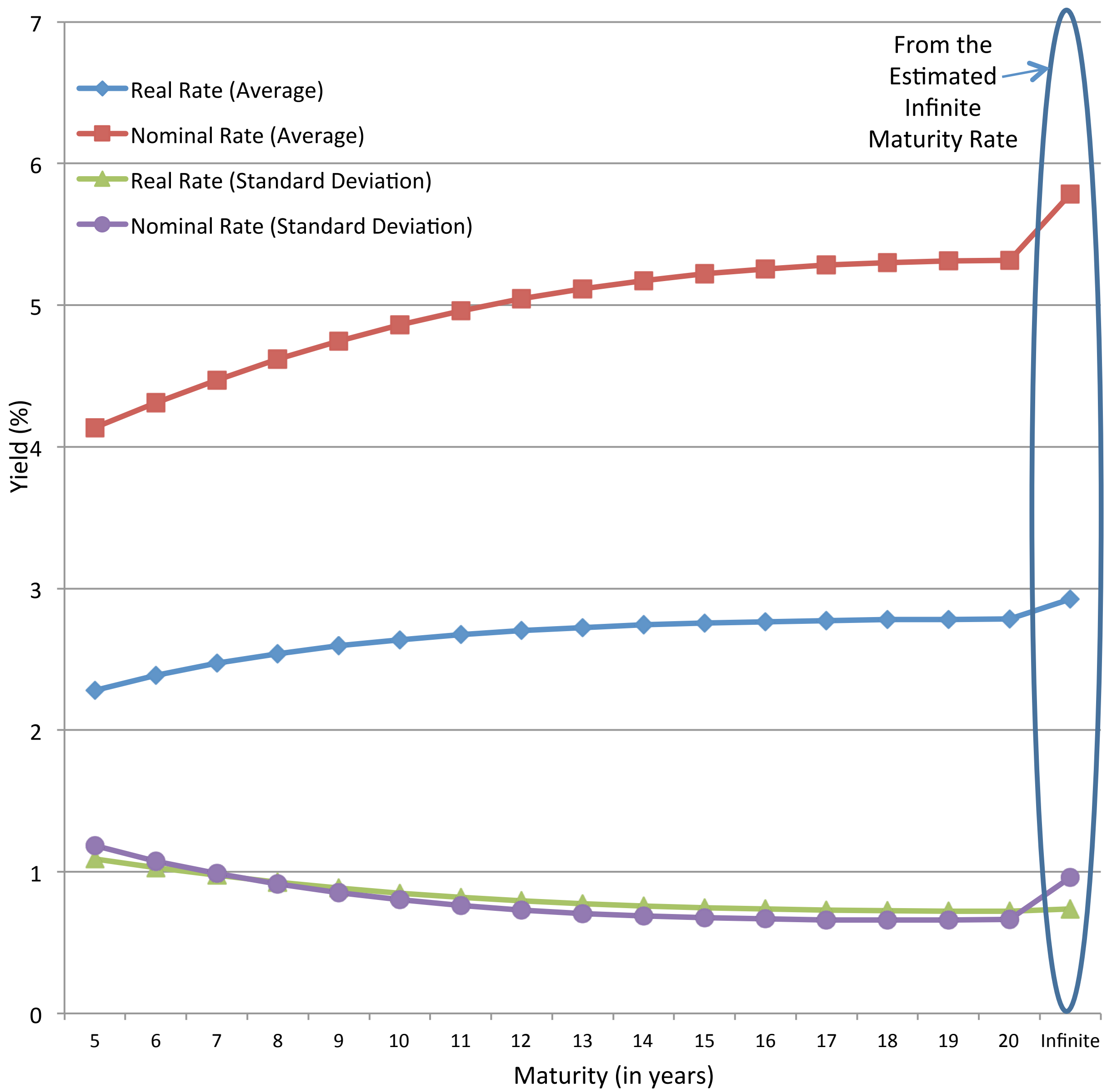

Figure 3 illustrates the averages and standard deviations of the zero coupon rate at each maturity from the original dataset spanning five years to 20 years maturity. These measures are displayed for both the TIPS and nominal Treasury. For comparison, the average and standard deviation of the CIR estimated infinite maturity zero coupon interest rates are displayed. As shown from original dataset, the average interest rate increases with maturity while the standard deviation of those rates decreases with maturity. Even if the infinite maturity interest rate is not stable, one might expect it to have a smaller standard deviation than those of shorter maturities, yet this is not the case. Compared to the 20 year maturity, the standard deviation of the asymptotic nominal rate increased while the standard deviation of the asymptotic TIPS rate remained similar to that of the 20 year rate. It appears that the estimated asymptotic rates do not vary less than those of shorter terms to maturity.

Our results also differ from those in Evans (1998) for the term structure of interest rates. In his estimates of the term structure of real interest rates in the U.K. from January 1984 until August 1995, Evans found that while the average nominal term structure was upward sloping, the real term structure was downward sloping. Our results indicate an upward sloping term structure for both nominal and real interest rates, although the real term structure is less steep. This difference may be due to the differing countries and/or the different time periods.

4. Conclusions

This paper empirically examines the long real interest rate and real interest rate term structure. We use monthly TIPS, for the period of 1999 to 2009, to predict infinite horizon forward real rates using a common one-factor term structure model and an adaptation of that model. Our results for this sample period show that, in the latter half of the period, implied long real rates fell substantially. From 1999 to 2004, the asymptotic real rates were relatively stable at three to four percent. After 2004, the asymptotic real rates fell to one to three percent. Additionally, we find that the standard deviation of asymptotic real rates is greater than that of shorter maturity real rates.

The period we examined experienced a decline in short-term interest rates. An examination of a period when short-term real interest rates rose may indicate that long rates increased during that period. Whether long real rates do increase during a period of rising short-term real rates is irrelevant to our conclusion since, according to term structure theory, long rates should never fall during any period.

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