An Information Economizing Approach to Capital Budgeting in Firms with Internal Capital Markets

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Abstract

Capital budgeters in large firms must match the supply of available funds to demand from worthy projects. Because detailed information about project values lies at the project group level, a centralized decision maker can find it difficult to determine the optimal budget size since different budget sizes have varying capital costs and hence changing individual project values. This paper introduces an information economizing capital budgeting procedure that allows decision makers to use duration information imbedded in the internal rate of return (IRR) to estimate the value of projects using a number of different discount rates. The sensitivity information is incorporated into an iterative procedure that allows a capital budgeter to start with an estimated budget size and converge to an optimal budget size. This procedure offers practical advantages over the traditional alternative of setting up an optimization problem or using Macaulay duration to adjust project values.

Key Words: Capital Budgeting, Duration, IRR, Internal Capital Markets
1. Introduction

Firms often operate their own internal capital markets to allocate resources efficiently. Project groups that independently develop or manage potential investments periodically submit information about candidate investments to a centralized decision maker who acts as a clearinghouse by matching supplies of funds, both external and internal, to worthwhile projects. Figure 1 depicts this arrangement. For example, a pharmaceutical company might have dozens of project groups, each working on molecules at various stages of development. At periodic reviews, these groups submit financial information about their projects to the central decision maker.

Scholars have commented that internal capital markets bring the financier closer to the investment, helping to reduce information asymmetries that typically affect traditional credit markets (Williamson, 1975; Myers and Majluf, 1984; Stein, 1997). Despite the fact that the financiers (capital budgeters) operate in the same firm as the project groups, detailed information about the value of investment opportunities resides at the project level in large and often complex organizations. For this reason, decision makers may not have all the information they need to make informed decisions. They rely on selected information submitted by project groups, often including net present value (NPV) and internal rate of return (IRR), to decide which projects to fund.

In order to calculate a project’s value, each project group must use an appropriate discount rate. The standard approach involves assuming that capital markets are perfect and therefore the firm’s cost of funding depends on the risk characteristics of the projects it undertakes, allowing the use of a risk-adjusted discount rate based on a project’s beta or some similar risk measure. As a practical matter, capital markets cannot perfectly evaluate the riskiness of individual projects internal to the firm and determining a project-specific discount rate can be problematic because most practitioners would have some difficulty compiling and analyzing the data required to estimate a project’s beta. Consequently, many companies use a firm-wide cost of capital derived from the beta of the company’s stock. Graham and Harvey (2001) find that 58.8% of surveyed firms always or almost always use a single company-wide discount rate and Bierman (1993) finds that 93% of Fortune 100 industrial firms surveyed use a company-wide discount rate. Thus in contrast to standard finance doctrine, capital markets are imperfect, making a company’s cost of funding a specific project insensitive to that project’s actual risk characteristics.

For companies using firm-wide discount rates, a problem arises because of constraints on the flow of information within the organization. Since a firm’s cost of funding typically increases with the amount of capital consumed, the cost of capital used to discount project cash flows should be based on the number of projects undertaken. However, the number of projects undertaken may not be known until all project values are submitted and the final projects selected. Because a shift in the discount rate can change the ranking of projects, this chicken versus egg problem presents a technical challenge to practitioners who seek more precision in their capital budgeting decisions.
The selection of projects by a centralized decision maker facing increasing funding costs can also be viewed and solved as an optimization problem (Weingartner, 1963). In the simplified case where the discount rate $r(k)$ is applicable to all time periods and is a function of the amount of capital required $k$, we would expect $r'(k) > 0$, reflecting that the cost of capital will increase as the amount of capital consumed increases. Formally, if we define $x_j$ as a decision variable that equals 1 if a project $j$ is undertaken and 0 if it is rejected, and if $I_j$ is the capital required for project $j$, the total capital required is $\sum(x_j I_j)$.

The capital budgeting problem can be formulated as:

$$
\text{Max } \sum_{x,k} NPV_j(r(k))x_j
$$

$$
s.t. \quad k > \sum_j x_j I_j
$$

$$
x_j \in (0,1)
$$

where $NPV_j(r)$ is the value of project $j$ discounted at $r$.

While this approach demonstrates some degree of technical sophistication, its practicality is limited. One issue is that it requires each project group to submit a spreadsheet with complete project cash flows to the centralized decision maker. Not only does this increase the possibility of error in data transmission or calculation, but some decision makers might view this approach as cumbersome. In addition, capital budgeters must outsource decision making to an optimization problem, which may not be acceptable to some
managers who view the process as a black box and prefer to make more straightforward evaluations based on NPV.

The alternative capital budgeting procedure presented in this paper specifies that project groups submit their project NPVs as well as the sensitivity of project values to changes in discount rates. This sensitivity factor will allow the centralized capital budgeter to determine the merit of either expanding or reducing the size of the budget. The sensitivity factor can take one of two forms. First, project groups could calculate and submit their projects’ Macaulay duration, which by definition is the sensitivity of investment value to interest rates. Alternatively, and as advocated in this paper, project groups can economize on already available information by submitting that project’s IRR, which has sensitivity information imbedded in it.

The centralized decision maker, starting with an estimated cost of capital (and associated budget size), can use the sensitivity information gleaned from IRR to converge to an optimal budget size by iterating through different discount rates. At each discount rate, the decision maker determines the NPV of the optimal allocation and the sensitivity of the NPV of the optimal portfolio to changes in the budget size. The sensitivity parameter ensures that as we iterate through different discount rates, at each step we improve the project portfolio’s value, ultimately converging to an optimal budget size.

Compared to the alternative of performing an optimization problem or computing the duration of cash flows, the information economizing approach uses familiar concepts making it more accessible to practitioners. Despite the fact that NPV is favored in academic circles, IRR is still widely used in industry (Fremgen, 1973; Mao, 1970; Ryan and Ryan, 2002; Graham and Harvey, 2001; Burns and Walker, 1997; Schall, Sundem and Geijsbeek, 1978). The information economizing approach described in this paper piggybacks on the widespread knowledge of IRR, to allow capital budgeters to adjust project NPVs for changes in discount rates without actually knowing the duration of a project’s cash flows.

2. Duration and IRR in Capital Budgeting

Our analysis begins with familiar assumptions, namely that expected project cash flows are known, the value of any one project is independent of the acceptance of any other project, and the same discount rate applies to each project and to all time periods. We also assume that all projects initially require cash expenditures and that the sign of the cash flows changes only once. This cash flow pattern results in what Teichroew, Robichek and Montalbano (1965) call pure investments. These pure investment-type projects will always have project values that are decreasing functions of their discount rates. We’ll assume that this function crosses the zero NPV line once, ensuring that we consider projects with only one IRR.

Many scholars have recognized the usefulness of duration to the capital budgeting decision (Blocher and Stickney, 1979; Boardman, Reinhart and Celec, 1982; Brown and Kulkarni, 1993; Cornell, 1999; Durand, 1974; Finch and Payne, 1996; Barney and
Danielson, 2004). Durand (1974), Boardman, Reinhart and Celec (1982) and Blocher and Stickney (1979) suggest that Macaulay duration can be used to find the sensitivity of project values to changes in discount rates. Traditionally, Macaulay duration, denoted below as $\Delta$, has been used to adjust the value of a stream of cash flows for changes in interest rates (Macaulay, 1938):

$$\Delta = \frac{\sum_{t=1}^{n} \frac{(t)CF_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}$$

where $r$ is the discount rate used to discount cash flows $CF$. Sensitivity of the value of a cash flow stream $NPV(r)$ to changes in $r$ is given by:

$$\frac{dNPV}{NPV} = -\Delta \frac{dr}{1+r}$$

The slope of the function $NPV(r)$ is:

$$\frac{dNPV}{dr} = -\Delta \frac{NPV}{1+r}$$

where $NPV$ is the value of the cash inflows minus the initial investment expenditure. This slope is shown in Figure 2.

Alternatively, using the information economizing approach, the capital budgeter approximates the slope of $NPV(r)$ using two points. The first point is $(r, NPV(r))$ and the second is $(IRR, 0)$. The line formed by these two points is shown in Figure 2. The estimated slope is:

$$\frac{dNPV}{dr} = m = \frac{-NPV(r)}{IRR - r} \quad (1)$$

Using this slope, the updated value of any project $NPV(r^{(2)})$ whose discount rate has changed from $r^{(1)}$ to $r^{(2)}$ is calculated as:

$$NPV(r^{(2)}) = NPV(r^{(1)}) + m(r^{(2)} - r^{(1)}) \quad (2)$$
Compared to the information economizing approach, the use of Macaulay duration to adjust project values has three drawbacks. First, practitioners do not fully understand the concept of duration and it is more difficult to calculate, making it harder for organizations to incorporate it into their capital budgeting processes. Second, for larger discount rate changes, the convexity of project values, much like the convexity of bonds, will make value adjustments using duration less accurate. Although Macaulay duration provides the most precise estimate of sensitivity to small discount rate changes, the information economizing approach can give better adjustments when changes are large and provide an exact estimate of value when the discount rate changes to the IRR. Finally, Macaulay duration becomes problematic for any but the most basic cash flow patterns (Durand, 1974). Strictly speaking, the duration of a project gives the value elasticity of the project’s cash inflows. This is not a problem for projects that have only a single cash outflow followed by a number of cash inflows because the value of the initial cash outflow is not sensitive to changes in the discount rate. However, when cash outflows are required in future years, the duration of these outflows must be subtracted from the duration of the cash inflows. Barney and White (2003) recognize this issue and suggest that managers compare the duration of a project’s operating cash flows to the duration of any project-specific financing.

In contrast to Macaulay duration, the information economizing procedure requires no additional information to be submitted to the decision maker beyond the project’s NPV and IRR, and is relatively easy for the decision maker to implement. While it is true that the information economizing approach will overstate project values somewhat for some
discount rates, this error is unavoidable for any linear estimate of a convex function, including Macaulay duration.

One might ask, which error is more acceptable – that imparted by Macaulay duration or by the information economizing approach? Although the answer to this question will depend on the size of the interest rate change, for most decision makers the information economizing approach should be preferable. The reason is that for capital allocators basing accept/reject decisions on NPV criteria, a change in the accept/reject decision requires a move in the discount rate past that project’s IRR. To say this another way, the size of the discount rate change that is critical for decision makers is the difference between the current discount rate and the IRR, which is exactly what is used in the two point information economizing approach. Note that the two point method gives good NPV fidelity when the cost of capital is near the IRR. Therefore, even though this method tends to overestimate a project’s NPV, it will not usually cause the capital budgeter to wrongly accept projects that actually have negative NPVs. By contrast, because the Macaulay duration method gives less precise NPV estimates near a project’s IRR, it may often lead a capital budgeter to reject positive NPV projects.

3. Projects with Multiple or No IRRs

Occasionally, a capital budgeter will encounter a project with multiple IRRs or no IRR. Projects with no IRR are usually either so obviously desirable or so undesirable that they may not merit thorough inspection. If they do, the approach outlined here will be of little help. On the other hand, when presented with multiple IRRs, a capital budgeter can sometimes glean information about discount rate sensitivity.

Figure 3 – Discount rate sensitivity of project value with multiple IRRs
Multiple IRRs can arise when a project switches between cash inflow and cash outflow more than once. These situations often present higher convexity, which can indicate that the slope of \( NPV(r) \) estimated from two points may not provide a good estimate of the slope for a wide range of discount rates. When presented with multiple IRRs, the capital budgeter’s best slope estimate will often be achieved by using the IRR closest to the initial discount rate estimate \( r^{(1)} \). As illustrated in Figure 3, if presented with IRRs of 7.16% and 33.67%, a capital budgeter with a cost of capital estimate of 10% would get a better local approximation of the slope by using 7.16% as the IRR for Equation (1). Likewise, if the cost of capital estimate is closer to 30%, then the IRR of 33.67% should be used. If the cost of capital estimate is near the midpoint of the two IRRs, it may indicate that \( NPV(r) \) is in transition from a negative to positive slope and the local sensitivity of NPV to discount rate changes is small. In any case, since exceptions to these general rules can be constructed, the analyst should proceed with caution when dealing with multiple IRR situations.

4. Iterative Optimization of Capital Budgets

A centralized capital budgeter is tasked with determining not only the size of the budget but also with choosing which projects should optimally comprise the project portfolio. This task is complicated by the fact that the cost of funding depends on the number of projects pursued, yet at the same time the value of each individual project depends on capital costs.

To begin, we define \( \pi \) as the maximum attainable NPV at a given budget size \( k \). Since each budget size has an associated cost of capital \( r \) that we assume is specified by the continuous and increasing function \( r(k) \), we can also describe \( \pi \) as a function of \( r \). Therefore, any mention of the sensitivity of \( \pi \) to some change in the size of the budget \( k \) is equivalent to the sensitivity of \( \pi \) to the change in \( r \) that is associated with some change in the budget size \( k \). To extend this relationship further, when in the capital budgeting procedure we describe iterating through various discount rates \( r \) in search of the highest value of \( \pi \), the reader may more intuitively interpret it as iterating through various budget sizes \( k \) in search of the one that yields the highest NPV.

At each step of the iterative procedure, we estimate the derivative of \( \pi \) with respect to \( r \). A positive slope indicates that total NPV can be increased by increasing the budget, and conversely, a negative slope means that some additional projects should be rejected. This can be viewed as an application of a gradient search algorithm (sometimes called “hill climbing” algorithm) over the univariate function \( \pi \). To facilitate the search for the optimal budget, we assume that \( \pi \) is a unimodal function, meaning that it has only a single peak. This assumption ensures that as we iterate through successively higher levels

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1 By assuming that \( r \) is an increasing function of \( k \), even when the budget is fully funded out of retained earnings and does not require external capital, we are assuming that the further reductions in the budget size will tend to build cash and hence reduce firm risk as perceived by investors.
of the function $\pi$, we ultimately arrive at a global maximum and not a local maximum for the function\(^2\).

To estimate the derivative of $\pi$, we consider that when $r$ increases, it impacts $\pi$ in two ways: a new projects effect resulting from the NPV of the additional projects added to the portfolio due to the increased budget size, and an existing projects effect resulting from the decrease in value of existing projects due to the higher discount rate. The new project effect can be estimated by finding the NPV of the next project that will be added by increasing the budget. The existing project effect is estimated by applying equations (1) and (2) to those projects that would be accepted at the current discount rate $r$. At any point, we can estimate $d\pi$ in the upwards direction by adding both the new projects effect and the existing projects effect that result from taking on one more project. Likewise, we can estimate $d\pi$ in the downwards direction by adding the loss of NPV that results from rejecting one additional project and the increase in NPV to existing projects due to the lower discount rate. When the change in the budget size shifts, the cost of capital from $r^{(n)}$ to $r^{(n+1)}$, the estimated change in $\pi$ is:

\[
\begin{align*}
    d\pi^+ &= NPV_{\text{new project}} + \sum_{\text{existing projects}} (r^{(n+1)} - r^n) m_{\text{existing}} \tag{3a} \\
    d\pi^- &= -NPV_{\text{removed project}} + \sum_{\text{existing projects}} (r^{(n+1)} - r^n) m_{\text{existing}} \tag{3b}
\end{align*}
\]

where $m$ is the sensitivity of an individual project’s NPV to changes in the discount rate as given by equation (1). Note that when we add a project as prescribed by equation (3a), the NPV of that project should be based on the latest discount rate estimate $r^{(n+1)}$. If the capital budgeter’s estimated NPV of the added project as submitted by the project group is based on a lower discount rate, that project’s value must be adjusted using equation (2).

At each budget size, we need to be able to determine which projects should comprise the portfolio. It turns out that finding the optimal portfolio and the resulting $\pi$ for a given budget is an easy problem to solve. For many years, it has been known that when operating under a fixed budget constraint, one maximizes NPV by ranking all positive NPV projects from highest to lowest by their profitability index (PI), allocating capital first to the highest PI project and subsequently moving on down the list until all funds are consumed (Lorie and Savage, 1955). The intuition is that since PI is defined as the ratio of the NPV of cash inflows to the NPV of cash outflows, the first projects accepted are

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\(^2\) In most cases, unimodality is a realistic assumption. To see why, consider that if $r$ increases enough, incrementally accepted projects will be less attractive and the discount rate used to value all projects will increase, resulting in lower $\pi$. Furthermore, as the budget decreases towards zero, very few projects are accepted and hence $\pi$ will be low. We might imagine that between these two extremes is a single optimal budget size that balances excessive capital costs with excessively diminutive budgets. This will certainly be the case if a company has many projects, none of which takes up a large portion of the available funding. If this is not the case, it is possible to construct an example that violates unimodality. For example, if a company has a number of small candidate projects as well as a number of larger ones, a graph of $\pi$ versus $r$ may have two peaks: one that includes only small projects and another that includes large projects. While such violations of unimodality are unlikely, they can cause the iterative approach to converge to a locally optimal budget rather than the true NPV maximizing budget.
those that give the highest NPV per dollar invested; therefore, PI is the proper decision rule to use when facing a fixed budget. Once the optimal project portfolio is identified for some budget level, \( \pi \) is simply the combined NPV of the accepted projects.

Although up until this point we have implicitly assumed that most investments require a single immediate cash outlay, a capital budgeter might encounter candidate projects that require expenditures spanning a number of years. The standard approach to handing multi-time period outflows is to calculate the present value of the investment’s cash outlays and use this as the required investment. While this approach will often work well, the capital budgeter may want to consider that planned future cash outlays may not always have the same impact on capital costs as current expenditures. For this reason, the acceptance of a project with an immediate expenditure of $1 million may increase the cost of capital more than a project with expenditures spread over 10 years with a present value of $1 million. One reason is that with multi-stage investment projects, a company may have the option of abandoning the project before spending the money in the future. This flexibility might make financiers more comfortable with increasing the size of the investment portfolio. Planned future expenditures will have a greater impact on current capital costs if lenders are required to make upfront capital commitments. In situations with multiple stages of investment, the capital budgeter should carefully consider how the acceptance of a project would increase capital costs and thus impact accepted project values.

5. Procedure

In the first step of the information economizing approach, the centralized decision maker provides a baseline cost of capital \( r^{(1)} \) to each project group. Second, each team calculates the NPV of its project using \( r^{(1)} \) and submits this information along with the IRR and the required investment for its project to the central decision maker. In the third step, the decision maker reviews all candidate projects and orders them according to PI, rejecting any with a PI of less than one, since these projects have a negative NPV. The highest PI projects are accepted first, then subsequently lower PI projects are added until all capital available at \( r^{(1)} \) is consumed. The sum of the NPVs of all accepted projects gives \( \pi^{(1)} \).

The next step is the true information economizing moment: the capital budgeter uses Equations (1), (2) and (3) to estimate the sensitivity of \( \pi \) at \( r^{(n)} \) (on the first iteration, \( n=1 \)) to changes in \( r \). If \( \pi \) is determined to be increasing in \( r \), then the capital budgeter should use his knowledge of \( r(k) \) to update the cost of capital from \( r^{(n)} \) to \( r^{(n+1)} \) (on the first iteration, \( r^{(1)} \) to \( r^{(2)} \)), to reflect the additional capital required to add one more project to the portfolio. In such a case, the project to be added is the one with the highest PI at \( r^{(n)} \). If \( \pi \) is decreasing in \( r \), then \( r^{(n+1)} \) should be lower than \( r^{(n)} \) based on similar reasoning. When decreasing the budget, the capital budgeter should remove the project with the lowest PI at \( r^{(n)} \) since as discussed in the previous section, it would yield the lowest NPV per dollar invested.
The centralized capital budgeter gives an estimated cost of capital $r^{(1)}$ to use when calculating NPV.

Each project group submits its IRR, required investment I and NPV based on $r^{(1)}$. Set $n=1$.

The capital budgeter determines the amount of capital available at $r^{(0)}$ (initially $r^{(1)}$) and accepts as many projects as can be funded at $r^{(0)}$ starting with those with the highest PI. The sum of the accepted projects’ NPVs gives portfolio value $\pi^{(0)}$.

The cost of capital is updated from $r^{(0)}$ to $r^{(n+1)}$ (for example, $r^{(1)}$ to $r^{(2)}$) by finding the cost of capital required to fund one additional project. Set $n=n+1$.

The sensitivity of $\pi$ to changes in $r$ is estimated using equation (1).

Is the portfolio value $\pi$ increasing, decreasing or insensitive to changes in $r$?

- Increasing:
  - $\pi$ is maximized. Allocate capital based on highest PI at $r^{(0)}$.

- Decreasing:
  - The cost of capital is updated from $r^{(0)}$ to $r^{(n+1)}$ (for example, $r^{(1)}$ to $r^{(2)}$) by finding the cost of capital required to fund one fewer project. Set $n=n+1$.

- Neither increasing nor decreasing:
  - The cost of capital is updated from $r^{(0)}$ to $r^{(n+1)}$ (for example, $r^{(1)}$ to $r^{(2)}$) by finding the cost of capital required to fund one fewer project. Set $n=n+1$.
At the new discount rate \( r^{(n+1)} \), the NPV of the optimal portfolio \( \pi^{(n+1)} \) is estimated using the sensitivity parameter given by equation (3). In changing to a different budget size, \( d\pi \) is added to \( \pi^{(n)} \) to get \( \pi^{(n+1)} \).

\[
\pi^{(n+1)} = \pi^{(n)} + d\pi
\]  

(4)

The process repeats at the new value of \( r \). A new sensitivity estimate is performed at \( r^{(n+1)} \) in the same way as before and a project is either added or subtracted from the portfolio depending on whether the sensitivity estimate indicates that the budget can be profitably expanded or cut. This process repeats until the capital budgeter arrives at a discount rate \( r^{(\infty)} \) and an associated \( \pi^{(\infty)} \), where the portfolio value \( \pi \) can no longer be increased by changing \( r \). At that point, both \( d\pi^+ \) and \( d\pi^- \) are negative and \( \pi \) is maximized.

Example

A company has an opportunity to invest in five projects, A, B, C, D and E, whose cash flows are shown in Table 1. Initially, the decision maker estimates a cost of capital of 10%, which is based on the cost of funding one project. The cost of capital for various budget levels is given in Table 2.

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Project A</th>
<th>Project B</th>
<th>Project C</th>
<th>Project D</th>
<th>Project E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-$1,000</td>
<td>-$1,000</td>
<td>-$1,000</td>
<td>-$1,000</td>
<td>-$1,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>$380</td>
<td>$200</td>
<td>$1,000</td>
<td>$650</td>
<td>$0</td>
</tr>
<tr>
<td>Year 2</td>
<td>$380</td>
<td>$250</td>
<td>$200</td>
<td>$650</td>
<td>$0</td>
</tr>
<tr>
<td>Year 3</td>
<td>$380</td>
<td>$300</td>
<td>$0</td>
<td>$650</td>
<td>$0</td>
</tr>
<tr>
<td>Year 4</td>
<td>$380</td>
<td>$600</td>
<td>$0</td>
<td>$650</td>
<td>$0</td>
</tr>
<tr>
<td>Year 5</td>
<td>$380</td>
<td>$600</td>
<td>$0</td>
<td>$650</td>
<td>$2,200</td>
</tr>
</tbody>
</table>

| NPV @ 10% | $440.50 | $396.19 | $74.38 | $1,464.01 | $366.03 |
| IRR     | 26.1%   | 21.6%   | 17.1%  | 58.5%     | 17.1%    |
| PI @ 10% | 1.44    | 1.40    | 1.07   | 2.46      | 1.37     |
| Sensitivity to \( r (m) \) | -$2,741.88 | -$3,409.97 | -$1,050.26 | -$3,018.41 | -$5,169.51 |

<table>
<thead>
<tr>
<th>Table 2 – Supply schedule for funds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required Funds (( k ))</td>
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<tr>
<td>Cost of Capital (( r ))</td>
</tr>
</tbody>
</table>

At the periodic review, the five project groups submit the IRR and NPV of their respective projects based on the capital budgeter’s cost of capital estimate of 10%. The capital budgeter ranks the projects based on PI as follows: D, A, B, E, C. Since at \( r=10\% \) funding is available for only one project, D, the project with the highest PI is chosen. \( \pi^1 \), the NPV of project D is $1,464.01.
Next the capital budgeter estimates the impact of increasing the budget beyond a single project. This upward sensitivity of $\pi$ to changes in $r$ involves the effect of adding project A (since it has the second highest PI) and reducing the value of project D due to the higher $r$. From equation (3a):

$$d\pi^+ = [NPV_A + (.12 -.10) * m_A] + [(.12 -.10) * m_D]$$
$$d\pi^- = [440.50 + (.12 -.10) * -2741.88] + [(.12 -.10) * (-3018.41)] = 325.29$$
$$\pi^2 = \pi^1 + d\pi^+ = 1464.01 + 325.29 = 1789.30$$

Notice that we also adjust the NPV of the added project (project A) for the 0.02 percentage point increase in the cost of capital. Next we estimate the sensitivity of $\pi$ to changes in $r$ at $r=0.12$. As before, this involves considering the additional NPV accrued from adding project B and the loss associated with higher capital costs.

$$d\pi^+ = [NPV_B + (.14 -.10) * m_B] + [(.14 -.12) * (m_D + m_A)]$$
$$d\pi^- = [396.19 + (.14 -.10) * -3409.97] + [(.14 -.12) * (-3018.41 - 2741.88)] = 144.58$$
$$\pi^3 = \pi^2 + d\pi^+ = 1789.30 + 144.58 = 1933.89$$

The next iteration involves estimating the value of expanding the portfolio to include project E.

$$d\pi^+ = [NPV_E + (.16 -.10) * m_E] + [(.16 -.14) * (m_D + m_A + m_B)]$$
$$d\pi^- = [366.03 + (.16 -.10) * -5169.51] + [(.16 -.14) * (-3018.41 - 2741.88 - 3409.97)] = -127.55$$
$$\pi^4 = \pi^3 + d\pi^+ = 1933.89 - 129.55 = 1806.34$$

At this point, the value of the portfolio declines with the addition of a fourth project, indicating that the optimal portfolio should include only three projects. Figure 4 is a plot of both the iterative estimates of $\pi$ and the actual estimates based on complete knowledge of the cash flows. Although the iterative procedure overestimates the portfolio value at any one point, it leads to the same capital allocation as an optimization based on complete knowledge of all cash flows.

Note that although in this example each project required the same initial $1,000 investment, this need not be the case. For example, if project A required an investment of $1,200 rather than $1,000, its profitability index would decrease from 1.44 to 1.20, making B (not A) the second project to be added. Hypothetically, if A did have the next highest PI, the change in $dr$ used in equations (3a) and (3b) to determine $d\pi$ would be slightly larger to reflect that additional $200 investment. In any case, as long as the capital budgeter has knowledge of $r(k)$, the iterative procedure can accommodate projects of all investment sizes.
6. Conclusion

IRR plays a special role in communicating information about the value of projects in organizations where information flow is constrained. The capital budgeting methodology introduced in this paper allows decision makers within decentralized organizations to adjust a project’s NPV for changes in discount rates using duration information imbedded in the project’s IRR. This approach highlights the usefulness of IRR for determining equilibrium in internal capital markets. Such an approach might be useful to companies whose discount rates are updated frequently or in situations where a large number of attractive projects increases total investment, and hence the marginal cost of capital.

The perspective advocated here aims to bridge capital budgeting theory to the realities of the modern organization. No decision maker has access to complete information; however, by not considering both IRR and NPV, one risks ignoring some useful information. The continuing disparity between theory and practice indicates that more scholarly work can be done to tailor capital budgeting to the context of the corporate decision maker.
References


