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A STATISTICAL COMPARISON OF VALUE AVERAGING VS. DOLLAR COST AVERAGING AND RANDOM INVESTMENT TECHNIQUES

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Abstract

As the title suggests, this paper compares two "formula" or mechanical investment techniques, dollar cost averaging and a relatively new proposal, value averaging, to a form of random investing to determine if any technique yields superior investment performance. Results indicate that value averaging does provide superior expected investment returns when investment prices are quite volatile and over extended investment time horizons with little or no increase in risk. These results are quite surprising based on other research supporting the Efficient Market Hypothesis and the fact that any actual performance attributed to value averaging does not result from any temporary inefficiency in market prices.

INTRODUCTION

In an earlier work Marshall and Baldwin [8] did a statistical comparison of Dollar-Cost Averaging (DCA) and random investment techniques. They calculated the internal rate of return (IRR) to an investor from each of many simulated investment scenarios under both techniques. They addressed the research question, "Does DCA yield superior investment performance compared to a purely random investment technique?" They found, with 99% confidence, that there is no statistical difference in the IRRs achieved by each technique. They also found, with 95% confidence that each technique had the same risk as measured by the standard deviation of the IRR distributions. They conclude that the null hypothesis is valid and that DCA is not superior to random investments. These results are contrary to most practitioner given investment advice and that presented in many texts on personal finance. See for example Gitman [5], Greene and Dince [6].

Clearly, the weak and semi-strong forms of the efficient market hypothesis (EMH) suggests that there should be few, if any, investment techniques that persist in giving meaningfully superior performance over time. Some techniques have given superior performance such as those investing in low P/E stocks, investing to take advantage of the "size effect" and even investing based on the January and other calendar related effects. See for example the work of Balvers, et. al. [1], Fama and French [4] and Rosenberg, et. al. [9]. However, if the market is efficient, as the hypothesis assumes, the benefits of such techniques should disappear as more and more investors participate in the anomalies.

What is interesting about tests of DCA and other purely mechanical techniques that are influenced only by the absolute level of the stock market and its subsequent fluctuation over time is that the corrective mechanism suggested by the EMH can not work since each investor may start using the mechanical technique at a different point in time and hence, at different stock price levels.

Edelson [2,3] has proposed another mechanical technique, somewhat similar to DCA, which he calls "Value Averaging" (VA). He has tested VA using simulations to compare VA to DCA and purchases of a constant number of shares in each investment period. Without considering possible differences in risk, Edelson [3, pp. 191 and 192] concludes:

- "(There is an) inherent return advantage of value averaging (over dollar-cost averaging and purchase of a constant number of shares)."
- "It's about as close to 'buy low, sell high' as we're going to get without a crystal ball."

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If Edelson is correct, and there are no compensating risk differences, then this is an important development. If so, he has discovered a mechanical anomaly that produces superior investment returns that is not dependent on temporary inefficiencies in the EMH. And, if VA "works", then additional research on other mechanical investment techniques that may be even better than VA should be encouraged.

Amazingly, no other published academic research other than Edelson's has tested Value Averaging. This research elects to correct that deficiency by testing the investment performance of VA against both DCA and random investment techniques. If Edelson is right VA should outperform DCA either on return or risk, or both. If Marshall and Baldwin are correct including VA in the 3-way test should show DCA and random investing delivering essentially equal performance based on IRR and risk assessment.

A DESCRIPTION OF VALUE AVERAGING

According to Edleson [3], the idea behind VA is simple. The investor sets a predetermined worth of the portfolio in each future time period, as a function of the size of the initial investment, the size of periodic investments and the yield expected. The investor then buys or sells sufficient "shares" or units of the investment such that the predetermined portfolio worth is achieved at each revaluation point. On yield expectation, Edleson [3, p. 119] suggests a long run equity return of 16%, based on a return 7.4% higher than the then existing rate on long term bonds. On revaluation timing, Edleson [3, p. 162] suggests that, "(using) value averaging two, three or four times a year would be reasonable...". In his own words, Edleson [2, p. 13] simply defines the value averaging concept:

> "The rule under value averaging is simple: ... make the value not (the market price) of your stock go up by a fixed amount each month."

Considering movements in the investment's market price and the size of periodic investments, the investor then either acquires or disposes of sufficient units of the investment such that the investment's required value is achieved at each subsequent revaluation point. During periods of market price decline, the investor is required to purchase relatively many units to maintain portfolio value. Conversely, during rising markets the technique requires the purchase of relatively few shares to achieve required value. During extended bull markets or during unusually large upward spikes in market price, the technique requires that units be sold to maintain portfolio value at the desired level.

This technique is even more intuitively appealing than DCA. As with DCA, more investment units are purchased when prices are low. However, VA requires that relatively more units be purchased as prices decline than does DCA since the unit price decline reduces the value of the portfolio . Furthermore, and contrary to DCA, VA gives a rule for selling. As the market price increases, beyond what it was recently, VA may require unit sales since the growing price rise increases the value of the portfolio. And, if the market price continues to increase dramatically, VA gives even more aggressive sell signals to control the value of the portfolio to the level desired. Marshall and Baldwin [8, p. 61] stated that DCA is appealing because,

> "Intuitively, DCA is contrary in the sense that fewer shares are purchased when price are 'high' and more shares are purchased when price are 'low', facilitating the 'buy low' aspect of the ancient investment adage, 'buy low, sell high'."

VA conceptually does an even better job. Even more units are purchased at "low" prices and probably some, at least, are sold at "high" prices.

At this stage, a numerical description of VA and a comparison to DCA may be useful. Table 1 shows that whether the market price of an investment is rising, falling, or fluctuating over time, VA yields a lower average cost of shares purchased than does DCA and both are lower than the average price of shares. While I am not enough of a mathematician to prove that this happens under all price patterns, the specific price patterns used are <u>not</u> selected solely to achieve this goal. The price patterns are the same ones used by Vanguard [11] to tout the benefits of DCA, and the same one used by Marshall and Baldwin [8] in their research. Interestingly, Vanguard has essentially discontinued mentioning the "benefits" of DCA subsequent to 1994.

The mathematical certainty (as reported by others, including Edleson [3, p. 30]) that DCA average cost is always lower than the average price has allowed some to promote DCA as an attractive way to assure superior investment performance. If that were sufficient to assure superior investment performance then by definition VA must be a superior to DCA since VA's average cost appears at least to be lower than DCA's. But, as demonstrated by Marshall

TABLE 1
Average Prices, Average Costs and IRRs for VA and DCA in Rising, Declining, and Fluctuating Markets

Rising Market

		Value Averaging				Dollar Cost Averaging		
Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest	Period Invest	Shares Bought	Shares Owned
1	$5	$400	80	80	$400	$400	80	80
2	8	800	100	20	160	400	50	130
3	10	1200	120	20	200	400	40	170
4	10	1600	160	40	400	400	40	210
5	$16	$2000	125	(35)	$(560)	$400	25	235
AVG:	$9.80				$600	$2000		
		Average Cost[1] : $4.80				Average Cost: $8.51		
		IRR: 33.83%				IRR: 32.01%		

Declining Market

		Value Averaging				Dollar Cost Averaging		
Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest	Period Invest	Shares Bought	Shares Owned
1	$16	$400	25	25	$400	$400	25	25
2	10	800	80	55	550	400	40	65
3	8	1200	150	70	560	400	50	115
4	8	1600	200	50	400	400	50	165
5	$5	$2000	400	200	$1000	$400	80	245
AVG:	$9.40				$2910	$2000		
		Average Cost : $7.28				Average Cost: $8.16		
		IRR: 24.08%				IRR: 24.80%		

Fluctuating Market

		Value Averaging				Dollar Cost Averaging		
Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest	Period Invest	Shares Bought	Shares Owned
1	$10	$400	40	40	$400	$400	40	40
2	8	800	100	60	480	400	50	90
3	5	1200	240	140	700	400	80	170
4	8	1600	200	(40)	(320)	400	50	220
5	$10	$2000	200	0	$0	$400	40	260
AVG:	$8.20				$1260	$2000		
		Average Cost : $6.30				Average Cost: $7.69		
		IRR: 15.22%				IRR: 13.15%		

1. Average cost can be calculated from the total of the "Period Invest" column divided by the number of shares owned at period 5. For example, using the Rising Market scenario, $600 total investment for VA bought 125 shares for an average cost of $4.80 a share, and $2000 total investment for DCA bought 235 shares for an average cost of $8.51.

TABLE 2

Average Prices, Average Costs and IRRs for VA and DCA in Rising, Declining and Fluctuating Markets Assuming a 10% Return for Value Averaging.

Rising Market

		Value Averaging				Dollar Cost Averaging		
Period	**Market Price**	**Value Required**	**Shares Owned**	**Shares Bought**	**Period Invest**	**Period Invest**	**Shares Bought**	**Shares Owned**
1	$5	$400.0	80.0	80.0	$400.0	$400	80	80
2	8	840.0	105.0	25.0	200.0	400	50	130
3	10	1324.0	132.4	27.4	274.0	400	40	170
4	10	1856.4	185.6	53.2	532.4	400	40	210
5	$16	$2442.0	152.6	(33.0)	$(527.6)	$400	25	235
AVG:	$9.80				$878.8	$2000		
		Average Cost : $5.76				Average Cost: $8.51		
		IRR: 33.89%				IRR: 32.01%		

Declining Market

		Value Averaging				Dollar Cost Averaging		
Period	**Market Price**	**Value Required**	**Shares Owned**	**Shares Bought**	**Period Invest**	**Period Invest**	**Shares Bought**	**Shares Owned**
1	$16	$400.0	25.0	25.0	$400.0	$400	25	25
2	10	840.0	84.0	59.0	590.0	400	40	65
3	8	1324.0	165.5	81.5	652.0	400	50	115
4	8	1856.4	232.1	66.6	532.8	400	50	165
5	$5	$2442.0	488.4	256.3	$1281.5	$400	80	245
AVG:	$9.40				$3456.3	$2000		
		Average Cost : $7.08				Average Cost: $8.16		
		IRR: -24.42%				IRR:-24.80%		

Fluctuating Market

		Value Averaging				Dollar Cost Averaging		
Period	**Market Price**	**Value Required**	**Shares Owned**	**Shares Bought**	**Period Invest**	**Period Invest**	**Shares Bought**	**Shares Owned**
1	$10	$400.0	40.0	40.0	$400.0	$400	40	40
2	8	840.0	105.0	65.0	520.0	400	50	90
3	5	1324.0	264.8	159.8	799.0	400	80	170
4	8	1856.4	232.1	(32.7)	(261.6)	400	50	220
5	$10	$2442.0	244.2	12.1	$121.0	$400	40	260
AVG:	$8.20				$1587.4	$2000		
		Average Cost : $6.46				Average Cost: $7.69		
		IRR: 16.22%				IRR: 13.15%		

and Baldwin [8], if there is no statistical difference in investment returns as measured by IRR between DCA and random investing, then logically, random investing must on average acquire shares at the same cost as DCA, time value of money considered. Therefore, the fact that VA acquires shares at lower average cost than DCA for these examples, or even in all cases, is not enough to assure that VA has a performance advantage over DCA.

The IRRs for both VA and DCA are shown in Table 1. Interestingly, but not necessarily statistically significant, VA has a higher IRR than DCA for each market price pattern shown. Some may argue that Table 1 is flawed. The "Value Required" column of VA is simply equal to the cumulative investment shown under the "Total Invest" Column of DCA, implying that the VA investor expects no return on investment. To counter that argument, to better match Edelson's methodology, and to further demonstrate the VA investment technique, Table 2 is presented.

This Table allows the "Value Required" column of VA to increase period to period by l0% of the prior period's "Value Required" plus the same $400 "Period Invest" shown for DCA, thus implying a 10% investment growth per period for VA. Again, the results are similar to Table 1. Each test shows VA with a lower average cost of shares than DCA and higher IRRs. However, the important question is not which technique yields the lower average cost of an investment. What really matters is the investment return achieved and the associated risk when a large number of comparisons are made.

METHODOLOGY

To more closely follow Marshall and Baldwin's [8] methodology, the three way analysis proposed (VA vs. DCA and Random) is structured similarly to their prior work. This analysis also provides a framework for considering the element of statistical risk, an area missing in Edleson [2,3]. The investment return of the three techniques is determined by the IRR of each simulation's cash flow. Five hundred simulations of investment results over time are used to calculate mean return and standard deviation of the IRR for a Base Case for each of the three investment techniques. The F-Test is used to test the variation among the three sample populations' mean IRR.

Twelve additional 500 run simulations are used to test if any variable considered by Marshall and Baldwin [8] has an effect on relative return. The same investment return criteria, IRR, is used in all analyses. The F-Test is calculated only for selected simulation cases where VA appears to have particular advantages.

To calculate each technique's cash flow pattern, the length of the investment time horizon, the dollar amount invested and the market price of the investment in each period are required. The IRR can then be calculated since the amount and timing of each periodic investment (or disinvestment) and the ending market value of the portfolio is known. For example, for a rising market as shown in Table 1, the "Period Invest" column under DCA requires a cash outflow of $400 each period 1 through 4. After a final investment of $400 in the fifth period, the DCA investor has acquired 235 shares with a market price of $16 a share for a total portfolio value of $3,760. The IRR of those cash flows is 32.01%, assuming annual time periods and if transaction costs and taxes are ignored.

A Base Case is prepared using a five year (actually 20 quarter) investment time horizon and price variability based on actual results achieved by the S&P 500 index during the period January 1, 1966 to March 31, 1989, randomly selected. This is the same historical period used by Marshall and Baldwin [8]. Investment amounts are held constant with DCA and the expected value of additional investments under random investing is equal to the amount of DCA investments. Under VA, investment amounts are determined by the VA algorithm and the actual price pattern randomly selected for each simulation. The Base Case is designed to be representative of price variability conditions experienced in the equity market in that period. Simulation gives us the ability to "live" the period 500 times instead of once.

Marshall and Baldwin [8] reported major differences among investment writers as to the appropriate conditions under which DCA was superior. See for example Greene and Dince [6] on market price variability or Sing [10] on market trend. Writers seem to suggest that market price variability, investment time horizon, variability of amount invested, market price trend and dollar amount invested all influence DCA investment performance, and we might conclude perhaps influence VA performance. Therefore, each such variable is incorporated into the three-way comparison by creating additional simulations.

Variability of the Amount of Investment

For VA, the amount invested each period is determined by the required value of the portfolio increasing by 10% each period plus the constant investment amount used for DCA. For DCA, a constant dollar amount invested each period, except in cases where the investment amount variability is tested. Random investing includes a 50% probability of investing in a particular period. And there is an equal remaining chance of investing either 150% or

250% of the amount invested with DCA. This random investing technique carries three advantages. First, it probably better approximates normal investment pattern such as “on/off” or “more/less” common among investors. Second, the probabilities assumed in the technique guarantee that the expected value of the investment is the same as in DCA. This way, we prevent a potential bias in our comparison by investing considerably more in one technique than the other. Third, it duplicates the method followed by Marshall and Baldwin [8], thus making comparison to their work easier.

Investment Time Horizon

A twenty quarter investment time horizon is assumed for the three techniques, except when the time horizon is tested. This is a long enough period to assure statistically significant results. A five year time horizon is also suggested by many investment writers, see for example Gitman [5]. A ten quarter and a forty quarter analysis are assumed when the effect of investment time horizon is tested. Again, Marshall and Baldwin’s [8] methodology is followed here and in the remainder of this section.

Market Price Variability

For the Base Case, changes in the quarterly S&P 500 Index (with dividends reinvested) between January 1, 1966 and March 31, 1989, is used to estimate historical market price volatility. The market price pattern for each simulation is randomly chosen from historical results in that period. That pattern is varied 500 times for each test based on the first difference of the total returns. The use of the quarterly first differences is one method to eliminate the upward trend that was experienced in the stock market in that period. We believe that this time period is sufficiently long and includes enough fluctuating markets so to be fairly representative of stock market variability.

Many investment writers, see for example Liscio [7] and Sing [10], disagree on the importance of the variability in investment’s market price in assuring DCA’s and perhaps VA’s superiority. Therefore, various other levels of variability are simulated to test the returns for DCA, VA and random investing. This variability effect is tested in two different ways. First, possible ±1%, ±5% and ±25% movements in the investment market price index per quarter are simulated. In each quarter, there is an equal probability of an increase or a decline in the market price by the percentages mentioned. Second, possible ±1, ±5, ±25 points movements in the index are tested in the same way. In both cases, the index is assumed to be 100 at the beginning of each simulation.

Market Price Trend

To test the importance of the underlying trend, cases are considered where the market price of the investment can both increase or decrease by 1.5% per quarter, not counting the variability effect mentioned previously. That means that with a twenty quarter investment time horizon, the expected market price will be almost 34.7% higher $(((1+.015)^{20}-1) \times 100\%)$ or 26.1% lower $(((1-.015)^{20}-1) \times 100\%)$ respectively at the end of the period when compared to the beginning. This assumption on underlying trend is not based on any particular period’s experience in the stock market, but the implied changes in market price are substantial and probably approximate the long-run return available in stocks (i.e. 1.5% a quarter plus dividends) as well as a significant decline in bear markets.

Dollar Amount of Investment

For some investors, and over a long-term investment time horizon, increasing investment funds may be available. Those progressing successfully in their careers will fit best into this category. On the other hand, some investors may need to invest less each period. Considering both possibilities, two different cases were studied where the dollar amount invested either increases or decreases by 2% per quarter.

Investment Return Analysis

Table 3, although unsophisticated statistically, is interesting. It shows how many times each technique was superior out of 500 simulation runs for each case tested. The highest IRR among the three investment techniques determines the winner of each simulation, with no regard as to size of the margin of victory.

TABLE 3
A Comparison of the Number of Times Each Investment Technique was Superior.*

	Value Averaging	Dollar Cost Averaging	Random Investing
Base Case	397	13	90
Variability (Per Quarter)			
± 1%	259	24	217
± 5%	324	23	153
±25%	422	20	58
Variability (Per Quarter)			
± 1 Point	254	22	224
± 5 Points	342	23	135
±25 Points	458	12	30
Market Trend (Per Quarter)			
+ 1.5%	393	19	88
- 1.5%	387	17	96
Investment Amount (Per Quarter)			
+ 2%	394	22	84
- 2%	406	20	74
Investment Horizon (In Quarters)			
10	318	21	161
40	421	18	61
Average Above	367.3	19.5	113.2
Percent of Total Simulations	73.5%	3.9%	22.6%

*A technique is defined as “superior” if it has the highest IRR, irrespective of the margin of “victory.”

To review, the Base Case assumes S&P 500 price variability, no market price trend, a constant investment amount for DCA, a 10% expected return on VA’s investment, a “random” amount invested in random investing and a 20 quarter investment time horizon. One variable at a time is changed in each subsequent case. For example, in the next case price variability is changed to ±1% per quarter instead of S&P 500 price variability with everything else being held the same. This routine is repeated for other measures of price variability, different market price trends, different investment amounts and different investment time horizons.

The results are very surprising in at least three ways. First, VA dominates both DCA and random investing. VA won all of the 13 tests by wide margins, 10 of those by more than doubling the next best technique, and VA won 73.5% of all simulations. Second, based on Marshall and Baldwin’s [8] results, it is surprising that DCA won so few times compared to random investing. In their research DCA won 48.2% of the time in head-to-head competition against random investing. Third, there is preliminary evidence for certain cases that seems to suggest that VA might actually produce higher investment returns. Any system that could improve investment returns would be favored both in a volatile investment price climate where it could work its “magic” and over longer time horizons where the benefits would compound. These effects are exactly described by certain results shown in Table 3, which are sorted and re-summarized in Table 4.

TABLE 4
A Comparison of the Number of Times Each Technique was Superior Under Various Sub-Sets Favoring, Disfavoring and Neutral on VA Investment Performance.

	Number of Times Each Technique was Superior		
	Value Averaging	**Dollar Cost Averaging**	**Random Investing**
Sub-Set 1 (Favorable)			
± 25% Variability	422	20	58
± 25 Point Variability	458	12	30
40 Quarter Horizon	421	18	61
Average Above	433.6	16.7	49.7
Percent of Total Simulations	86.8%	3.3%	9.9%
Sub-Set 2 (Unfavorable)			
± 1% Variabililty	259	24	217
± 1 Point Variability	254	22	224
10 Quarter Horizon	318	21	161
Average Above	277.0	22.3	200.7
Percent of Total Simulations	55.4%	4.5%	40.1%
Sub-Set 3 (Neutral)			
+ 1.5%/Quarter Market Trend	393	19	88
- 1.5%/Quarter Market Trend	387	17	96
+ 2.0%/Quarter Investment Amount	394	22	84
- 2.0%/Quarter Investment Amount	406	20	74
Average Above	395.0	19.5	88.5
Percent of Total Simulations	79.0%	3.9%	17.1%

The three cases included in Sub-Set 1 of Table 4 give VA all the benefits of a high price variability and long investment horizon. Here VA totally dominates DCA and random winning more than 6.5 times more often than the other two combined. Conversely, the three cases included in Sub-Set 2 give VA the least benefits of volatility and time and while VA still wins more often, as would be expected of any superior technique, the margin of victory declines dramatically, to less than 1.2 times the other two combined. Interestingly, these results on volatility agree with Edleson [3, pp. 142-144] but are in conflict with this finding on time horizon [3, pp. 144-146]. Finally, it is difficult to see how the buy/sell rules implicit in VA could be favored by the overall direction of the market or by the investment amount pattern of a particular investor. Notice that whether the market trend is up or down and the amount invested grows or declines with time, the VA "win" ratio remains essentially constant in Sub-Set 3 and is very similar to results shown for the Base Case in Table 3.

Interesting as all that may be, to be sure that VA really does provide superior IRRs, a more statistically sophisticated analysis is required. Table 5 shows mean IRRs for each of the three techniques with F-Test scores for selected simulation cases.

The mean IRRs for VA are without exception higher than either DCA or random investing, and overall VA outperforms both DCA and random investment techniques by almost three percentage points. Also interesting is that DCA and random investing yield essentially the same mean IRRs (-1.14% and -1.11% respectively) even though the random investment technique dominated DCA in the number of times it was superior (22.6% vs. 3.9% respectively) as shown in Table 3. This comparison of mean IRR of DCA versus random investing is a confirmation of the work of Marshall and Baldwin [8].

TABLE 5
Mean IRRs (%) for VA, DCA, and Random Investing and Selected F-Test Scores*

	Value Averaging	Dollar Cost Averaging	Random Investing	F-Test Score**
Base Case	0.50	(1.21)	(1.10)	3.40
Variability (Per Quarter)				
± 1%	(0.10)	(0.11)	(0.11)	0.02
± 5%	(0.53)	(0.81)	(0.81)	
±25%	(2.87)	(9.93)	(9.96)	12.04
Variability (Per Quarter)				
± 1 Point	(0.18)	(0.19)	(0.19)	
± 5 Points	(1.01)	(1.35)	(1.36)	
±25 Points	24.90	5.69	5.83	
Market Trend (Per Quarter)				
+ 1.5%	6.71	4.83	4.96	
- 1.5%	(5.44)	(6.99)	(6.89)	
Investment Amount (Per Quarter)				
+ 2%	0.29	(1.16)	(1.20)	
- 2%	0.44	(1.12)	(1.18)	
Investment Horizon (In Quarters)				
10	0.72	(0.68)	(0.76)	1.50
40	(0.34)	(1.84)	(1.71)	4.70
Average Above	1.78	(1.14)	(1.11)	
Advantage to VA	N/A	2.92	2.89	

*Notice the large number of negative IRRs. First, recall that these simulations do not reflect any particular historical investment period and therefore can not be judged against returns actually achieved. Second, the tests apply a variability pattern from S&P 500 returns that is essentially a normal distribution for all simulations not testing variability. When variability is tested equal percentage or point probabilities of rising or falling market prices are used. The latter effect and stochastic calculus skews returns to less than the expected 0% IRR for cases not involving an implicit upward or downward market trend. In the simplest terms, a -5% return, from which you need a +5.26% return to recover to the previous position, is "bigger" than a +5% return.

**At a confidence level of 99%, F is significant at 2.33.

Table 5 also shows the F-test score for the base case and four other selected simulations of particular interest to VA's performance. With 99% confidence, the mean IRRs differ for F-Test scores greater than 2.33. Three of the five simulations--the Base Case, the case with price variability of ±25% per quarter and the case with a 40 quarter investment horizon, exceed that F-Test score. Therefore, for those cases, there is a 99% certainty that the mean IRRs of the three investment techniques are statistically different. The inference must be that VA's IRR is statistically larger than either DCA or random techniques, in agreement with Edleson's findings.

As mentioned previously, any system that could improve investment returns would be favored by a volatile investment climate and over a long time horizon. Table 6 compares mean IRRs for each technique under favorable, unfavorable, and neutral conditions as did Table 4. Table 6 sorts and re-summarizes data from Table 5.

TABLE 6
A Comparison of the Mean IRRs of Each Technique Under Various Sub-Sets Favoring, Disfavoring and Neutral on VA Investment Performance.

	Mean IRR (%)		
	Value Averaging	**Dollar Cost Averaging**	**Random Investing**
Sub-Set 1 (Favorable)			
± 25% Variability	(2.87)	(9.93)	(9.96)
± 25 Point Variability	24.90	5.69	5.83
40 Quarter Horizon	(0.34)	(1.84)	(1.71)
Average Above	7.23	(2.03)	(1.95)
Advantage of VA	N/A	+9.26	+9.18
Sub-Set 2 (Unfavorable)			
± 1% Variabililty	(0.10)	(0.11)	(0.11)
± 1 Point Variability	(0.18)	(0.19)	(0.19)
10 Quarter Horizon	(0.72)	(0.68)	(0.76)
Average Above	(0.15)	(0.33)	(0.35)
Advantage of VA	N/A	+0.48	+0.50
Sub-Set 3 (Neutral)			
+ 1.5%/Quarter Market Trend	6.71	4.83	4.96
- 1.5%/Quarter Market Trend	(5.44)	(6.99)	(6.89)
+ 2.0%/Quarter Investment Amount	0.29	(1.16)	(1.20)
- 2.0%/Quarter Investment Amount	0.44	(1.12)	(1.18)
Average Above	0.50	(1.11)	(1.08)
Advantage of VA	N/A	+1.61	+1.58

Again, VA performs as though it truly is a technique that generates higher IRRs. As required under favorable conditions (Sub-Set 1) it generates IRRs about 9 percentage points higher than DCA or random. Under unfavorable conditions (Sub-Set 2) VA's advantage declines to about one-half of a percentage point. Under neutral conditions (Sub-Set 3) VA's advantage is about 1.5 percentage points, in between the other Sub-Sets. Clearly, VA's performance advantage is small in neutral and unfavorable investment environments, and perhaps not even statistically provable. However, under the favorable conditions of high volatility and with a long investment horizon, VA's nine plus percentage point advantage is impressive.

F-Test results calculated for the Base Case, high and low volatility and short and long investment horizon show with 99% certainty that VA's IRRs are statistically different than DCA's and random investment's IRRs for high variability and long investment time horizon, as well as for the Base Case. No such claim can statistically be made for low variability and short investment time horizon. Logically though, if VA does produce higher IRRs in a favorable environment it should also produce higher IRRs in all environments, at least on an expected value basis.

INVESTMENT RISK ANALYSIS

Table 7 shows the standard deviations of mean IRRs calculated for each simulation, as well as the average standard deviation for all simulations. The standard deviations of mean IRR for each investment technique are quite comparable for each case simulated. The biggest difference shown is in the ±25% per quarter price variability case,

TABLE 7
Standard Deviation (%) of Mean IRRs.*

	Value Averaging	Dollar Cost Averaging	Random Investing
Base Case	11.77	11.35	11.66
Variability (Per Quarter)			
± 1%	1.03	1.02	1.05
± 5%	5.27	5.18	5.32
±25%	27.09	24.63	25.00
Market Trend (Per Quarter)			
+ 1.5%	12.28	11.93	12.30
- 1.5%	11.21	10.76	11.09
Investment Amount (Per Quarter)			
+ 2%	11.91	11.69	11.95
- 2%	11.70	11.24	11.54
Investment Horizon (In Quarters)			
10	14.97	14.86	15.70
40	8.70	8.33	8.40
Average Above	11.59	11.10	11.40
Disadvantage to VA	N/A	0.49	0.19

*Some readers may wonder whether the generally negative IRRs shown on Table 5 and the substantial standard deviations shown here imply a logic or methodology problem since those results are not consistent with the Capital Asset Pricing Model where increased investment risks should be rewarded by higher expected returns. I believe not. The methodology does not reproduce any particular historical pattern of investment prices over time where risk should be compensated. This paper simply tests for relative performance of VA versus DCA and random investing. Except for the mathematical effect mentioned in the footnotes to Table 5, I would in fact expect an average IRR of 0% except in the case where an upward or downward market trend is applied.

where the 27.09% standard deviation for VA exceeds the 24.63% standard deviation for DCA by less than 10%. On average, VA had the highest standard deviation (11.59%) and DCA had the lowest (11.10%)--less than a 5% difference.

The correct question is, does a statistically significant difference in standard deviation exist among the three techniques? In their earlier work, Marshall and Baldwin [8] used the Chi Square test on the base case to test the shape of the distribution of the IRRs in each simulation. They concluded, with 95% confidence, that both DCA and random investment techniques' IRRs were drawn from the same normal distribution and, therefore, could not differ in risk. That statistical test was based on a standard deviation of 11.19% for DCA and 11.37% for random investing using the identical cases and methodology. These results (11.77% for VA, 11.35% for DCA, and 11.66% for random investing) are so close to those reported in Marshall and Baldwin [8] that there should be confidence about inferring that there is no statistically meaningful difference in risk among VA, DCA and random investment techniques.

A FINAL TEST

After continually being amazed by VA's performance in theoretical tests, a test of results in the real world of stock price movements must be made. Table 8 shows a comparison of the performance, measured again by IRR, of each of the three methods, using historical S&P 500 total returns quarter over the period January 1, 1996 through March 31, 1989.

TABLE 8
IRRs(%) for VA, DCA, and Random Investing Using Actual S&P 500 Index Price Trends Over the Periods Shown

20 Quarter Investment Periods Beginning	Value Averaging	Dollar Cost Averaging	Random Investing
1Q 1966	5.20	3.94	4.25
2Q 1971	5.56	3.83	3.04
3Q 1976	4.47	3.53	3.95
4Q 1982	(7.23)	(12.13)	(7.86)
1Q 1984	13.72	11.33	11.82
Entire Period 1Q 1966 - 1Q 1989	7.58	7.47	7.01

For each of the five periods tested VA had a higher IRR than DCA or random investing. For the entire period VA also won. Based on thousands of simulations, VA wins. Theoretically, based on improved performance in volatile markets and over long time periods, VA wins. Historically, VA wins. The evidence that VA does produce superior performance seems strong.

REASONS THAT REPORTED RESULTS MAY UNDERSTATE THE BENEFITS OF VALUE AVERAGING

As pointed out by Edleson [3, pp. 176-177] implementing VA requires the use of a "side fund". This is essentially a money market fund into which periodic savings are deposited prior to investment. The side fund also allows for the accumulation of money from VA's sell signals and provides the funds to implement VA's buy signals in amounts exceeding periodic savings. Although there is a small lag between periodic savings and the implementation of VA, (recall, Edleson revalues "three, four or five times a year") it is likely that over a reasonable time horizon, the side fund will experience periods when a not insignificant portion of the investor's portfolio will be in the money market fund rather than invested in the market. Such a period would be similar to 1998 for investors that implemented VA shortly after Edleson's [3] book was published.

This paper ignores both the return available from temporary investments in the side fund and the reduction in overall portfolio risk inherent in a money market fund. If both were included, the return advantage to VA over both DCA and random investing would improve and the risk would be reduced. The magnitude of such adjustments is likely to be small, depending primarily on the average percentage of portfolio allocated to the side-fund, but the direction of the change unarguably favors VA.

CONCLUSIONS

Results strongly suggest, believe it or not, that value averaging does actually provide a performance advantage over dollar-cost averaging and random investment techniques, without incurring additional risk. As might be expected from a technique that does outperform, the higher the price variability and the longer the investment time horizon the better. Each gives value averaging the time and the opportunity to work its "magic". The results are amazing and Dr. Edleson should be congratulated on seemingly important work in developing value averaging. However, peer review of this work and other tests of value averaging are important steps to confirm or refute these findings. Finally, results also suggest that there is no statistical difference between DCA and random investment techniques either in expected return or in risk avoidance, thus confirming the earlier work of Marshall and Baldwin [8].

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