

## **ESTIMATING SYSTEMATIC RISK: THE CHOICE OF RETURN INTERVAL AND ESTIMATION PERIOD**

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### **Abstract**

Capital budgeting is one of the most important strategic decisions that face financial managers. It is the process where the firm identifies, analyzes, and selects long-term projects. One popular technique to evaluate whether the project should be undertaken is the net present value method. A key ingredient of net present value is an accurately estimated weighted-average-cost-of-capital that is used to discount the future cash flows. The most difficult component of the weighted-average-cost-of-capital to calculate is the cost of equity. One approach to estimate the cost of equity is the Capital Asset Pricing Model approach where the financial manager estimates the firm's beta. A time-series regression is often used to estimate the beta and requires the financial manager to select both a return interval and an estimation period. This study examines the return interval and estimation period the financial manager should select when estimating beta. The results show that the financial manager should select the daily return interval and an estimation period of three years or less.

### **INTRODUCTION**

Capital budgeting is one of the most important strategic decisions that face financial managers. It is the process where the firm identifies, analyzes, and selects long-term projects. One popular technique to evaluate whether the project should be undertaken is the net present value (NPV) method. A key ingredient of the NPV method is an accurately estimated weighted-average-cost-of-capital (WACC) that is used to discount the future cash flows. To estimate the WACC, the financial manager must calculate the after-tax cost of debt, the cost of preferred stock, and the cost of equity. The most difficult component to calculate is the cost of equity. One method to calculate the cost of equity is the Capital Asset Pricing Model (CAPM), which is examined in this paper.

The fundamental premise of the CAPM is that the risk of a stock can be decomposed into two components. The first component is systematic risk, which is related to the overall market. The second component is non-systematic risk, which is specific to the individual stock. The CAPM approach further asserts that the expected return for a security is related only to the security's beta, which is the measure of systematic risk.<sup>1</sup> Unfortunately, financial managers cannot directly observe beta, but must estimate it. To estimate the beta of a firm, a time-series regression is often used and requires the financial manager to select both a return interval and an estimation period.

An individual firm requires great precision in the individual estimate of beta. Large standard errors of the estimated beta increase the uncertainty surrounding the cost of equity which could have unintended consequences in capital budgeting decisions. Thus, financial managers face a dilemma when estimating beta. The more observations used in the estimation period the smaller the standard error of the estimated beta, which improves its precision. However, adding more observations also lengthens the calendar time of the estimation period, which increases the likelihood that structural characteristics of the firm have changed, leading to a change in beta. For example, during the last decade many firms recapitalized, acquired divisions, spun-off divisions, or substantially changed product mix. Any of these actions could change the beta or systematic risk of the firm.

This paper examines four return intervals and eight estimation periods to determine what return interval and estimation period financial managers should use to estimate beta. Previous studies give little insight into the choice

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of the time interval over which a return is measured. Although most current empirical research use daily returns, previous studies examine weekly, monthly, quarterly, and annual returns. Regarding the beta estimation periods, previous research has recognized this problem and has attempted to address it.<sup>2</sup> This body of research concludes that the optimal estimation period ranges from four to nine years. From the financial manager's viewpoint, he wants the period as long as possible to increase the precision of the beta estimate, but not too long such that the beta estimate is likely to be biased and of little value.

In contrast to previous studies, the results show that for a given estimation period, daily returns provide a smaller standard error of the estimated beta than do weekly, two-weekly, or monthly returns. Thus, the financial manager should use the daily return interval to estimate beta because it increases its precision. This paper also concludes that a much shorter estimation period, two to three years, is more appropriate for financial managers to use when estimating beta with daily returns. The remainder of this study is organized as follows. The next section discusses the data and empirical tests. The last section provides a brief summary.

## EMPIRICAL TESTS

### Methodology and Data

When using the CAPM approach to calculate the cost of equity, the financial manager must select the return interval and the estimation period to be used to estimate beta. This study examines four return intervals, which include daily, weekly, two-weekly, and monthly returns. It also examines eight estimation periods ranging from one year to eight years over the 1982 to 1989 period.

Security returns are obtained from the CRSP NYSE/AMEX databases. The first sample of firms is selected from the daily database. The daily returns are then compounded to create two additional samples, one of weekly returns (Friday to Friday) and the other of two-weekly returns. There are 1329 firms in each of these three samples. The CRSP NYSE/AMEX monthly database is used to create a fourth sample. There are 946 firms in the fourth sample.<sup>3</sup>

The following market model is used to estimate the beta of a firm:

Equation 1

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

where  $R_{it}$  is the stock return on firm  $i$  in period  $t$ ,  $R_{mt}$  is the CRSP equally-weighted market return in period  $t$ ,  $\alpha_i$  is the intercept,  $\beta_i$  is the beta for firm  $i$ , and  $\varepsilon_{it}$  is the error for firm  $i$  in period  $t$ . The standard error of the estimated beta is denoted as  $S_\beta$  and is defined as:

Equation 2

$$S_\beta = 1/(N-1)^{0.5} \times (S_\varepsilon / S_m)$$

where  $S_\varepsilon$  is the standard deviation of the estimated errors in equation (1),  $S_m$  is the standard deviation of the market returns, and  $N$  is the number of observations.

### Simulations Based on Assumptions of Stationarity

For the moment, assume firms do not recapitalize, acquire and spin-off divisions, or change their product mix. Thus, the structural characteristics of the firm do not change and we can assume that the beta of firms is stationary for eight years or that  $S_\varepsilon$  and  $S_m$  are constants. Using typical values of  $S_\varepsilon$  and  $S_m$ , how many observations are sufficient to capture most of the precision due to added observations?

Using daily returns, equation (1) is estimated for each security in each year of the study period with the  $S_\varepsilon$  saved for each regression. Next the mean  $S_\varepsilon$  is calculated for the sample. Then the  $S_m$  is computed for each year. This process is repeated for each of the other return intervals. Table 1 reports the  $S_m$  and the mean  $S_\varepsilon$  for each year and each return interval. The mean  $S_\varepsilon$  is stable for each return interval and year. The  $S_m$  is also stable for each return interval and year, with the exception of 1987.

**TABLE 1**  
**Estimates of  $S_e$  and  $S_m$  using daily, weekly, two-weekly,**  
**and monthly returns for each year from 1982 to 1989.**

Year	Return Interval							
	Daily		Weekly		Two-Weekly		Monthly	
	$S_m$	Mean of $S_e$	$S_m$	Mean of $S_e$	$S_m$	Mean of $S_e$	$S_m$	Mean of $S_e$
1989	0.00477	0.02156	0.01221	0.04301	0.01541	0.05676	0.03121	0.06828
1988	0.00632	0.02148	0.01343	0.04305	0.02113	0.05620	0.03459	0.06920
1987	0.01608	0.02560	0.03370	0.04885	0.05846	0.06321	0.09258	0.07719
1986	0.00629	0.02216	0.01723	0.04639	0.02595	0.06207	0.04556	0.07594
1985	0.00459	0.01964	0.01306	0.04087	0.02336	0.05446	0.04038	0.06682
1984	0.00610	0.02021	0.01815	0.04118	0.02615	0.05494	0.04477	0.06831
1983	0.00667	0.02158	0.01674	0.04658	0.02706	0.06271	0.03433	0.07615
1982	0.00840	0.02390	0.02517	0.04954	0.04253	0.06512	0.05780	0.07536
Mean	0.00740	0.02202	0.01871	0.04493	0.03001	0.05943	0.04765	0.07216

$S_e$  is the estimated standard deviation of the errors from equation (1).

$S_m$  is the standard deviation of the market return.

The average value of the  $S_e$  and the  $S_m$  during the eight-year period is used to simulate values of  $S_\beta$  for different estimation periods and different return intervals. Table 2 reports the average value of the  $S_e$  and the  $S_m$  as well as the simulated  $S_\beta$  for each estimation period and return interval. For a short estimation period such as one year of daily returns,  $S_\beta$  is quite large with a 95 percent confidence interval of 0.62 to 1.38 for a firm with an estimated beta of 1.0. If the estimation period increases to two years of daily returns, then the  $S_\beta$  is much smaller with a 95 percent confidence interval of 0.73 to 1.27 for a firm with an estimated beta of 1.0. If the estimation period increases to three years of daily returns, then the  $S_\beta$  is still much smaller with a 95 percent confidence interval of 0.78 to 1.22 for a firm with an estimated beta of 1.0. If the estimation period increases to eight years of daily returns, then the  $S_\beta$  is very small with a 95 percent confidence interval of 0.87 to 1.13 for a firm with an estimated beta of 1.0.

As the return interval increases from daily to weekly to two-weekly to monthly, the ratio of  $S_e/S_m$  decreases from 2.9757 to 1.5144. This indicates that the longer return interval smoothes out some of the noise in the return generating process. However, for any given estimation period, the daily return interval always provides a more precise estimate of beta as measured by the  $S_\beta$ . In other words, the increased number of returns associated with a daily return interval more than compensates for the additional noise of the shorter return interval.

Suppose a financial manager decides the minimum estimation period is one year and the maximum estimation period is eight years. Additionally, the  $S_\beta$  for one year of daily returns is 0.1886 and for eight years of daily returns is 0.0665. Thus, the maximum reduction in the  $S_\beta$  for increasing the estimation period from one year to eight years is 0.1221. Note that increasing the estimation period from one year to two year captures about 45 percent of the maximum reduction as the  $S_\beta$  decreases from 0.1886 to 0.1332. Increasing the period to three years and four years captures about 65 and 77 percent, respectively.

There are several conclusions can be drawn from these simulations. First, given identical return periods, shorter return intervals are associated with smaller  $S_\beta$  or greater precision in estimating beta. Thus, financial managers should use daily data to estimate beta, if given a choice. Second, increasing the estimation period from one year to three years substantially decreases the  $S_\beta$  or increases in the precision of the estimated betas. In fact, the three-year estimation period captures 65 percent of the maximum reduction obtainable by increasing the estimation period from one year to eight years.

**TABLE 2**

Simulated mean standard errors of estimated betas as a function of the estimation period and return interval.  $S_e$  and  $S_m$  are overall means from Table 1 and are based on 1982 to 1989 data.

$$S_\beta = 1/(N-1)^{0.5} \times (S_e / S_m)$$

Estimation Period (Years)	Return Interval							
	Daily ( $S_e/S_m=2.9757$ )		Weekly ( $S_e/S_m=2.4014$ )		Two-Weekly ( $S_e/S_m=1.9803$ )		Monthly ( $S_e/S_m=1.5144$ )	
	Returns	$S_\beta$	Returns	$S_\beta$	Returns	$S_\beta$	Returns	$S_\beta$
1	250	0.1886	52	0.3330	26	0.3884	12	0.4372
2	500	0.1332	104	0.2355	52	0.2746	24	0.3091
3	750	0.1087	156	0.1923	78	0.2242	36	0.2524
4	1000	0.0941	208	0.1665	104	0.1942	48	0.2186
5	1250	0.0842	260	0.1489	130	0.1737	60	0.1955
6	1500	0.0768	312	0.1360	156	0.1586	72	0.1785
7	1750	0.0711	364	0.1259	182	0.1468	84	0.1091
8	2000	0.0665	416	0.1177	208	0.1373	96	0.1546

$S_e$  is the estimated standard deviation of the errors from equation (1).

$S_m$  is the standard deviation of the market return.

## Estimation Based on Actual Data

This section analyzes whether actual data corroborates the simulated results above. Consider a financial manager whose objective is to estimate the current beta for a particular firm. The financial manager must select both the return interval and the estimation period, which would begin at the current date and extend into the past for some desired number of years. The following tests are designed with this process in mind.

Using daily returns, each firm's beta is estimated eight times. The first beta is estimated with one year of daily returns, the 1989 data; the second beta is estimated with two years of daily returns, the 1988 to 1989 data. The other six betas are estimated in a similar manner, with the final beta is estimated with eight years of daily returns, the 1982 to 1989 data. For each beta, the standard error of the estimated beta,  $S_\beta$ , is saved as a measure of the precision in estimating systematic risk. This procedure is then repeated for each firm using the other three return intervals of weekly, two-weekly, and monthly.

Table 3 reports the mean  $S_\beta$  for each return interval and estimation period.<sup>4</sup> If one year of daily returns are used to estimate beta, then the mean  $S_\beta$  is 0.2826. If two years of daily returns are used to estimate beta, then the mean  $S_\beta$  decreases to 0.1723. In fact, as the estimation period increases, the mean  $S_\beta$  continues to fall, with the minimum mean  $S_\beta$  occurring in the eight-years estimation period with a  $S_\beta$  of 0.0621.<sup>5</sup> Thus, the maximum reduction in the  $S_\beta$  for increasing the estimation period from one year to eight years is 0.2205. Note that increasing the estimation period from one year to two year captures about 50 percent of the maximum reduction as the  $S_\beta$  decreases from 0.2826 to 0.1723. Increasing the period to three years captures about 91 percent of the maximum reduction. Thus, longer estimation periods are associated with very small improvements in the precision of the estimated beta.<sup>6,7</sup> The other return intervals results are similar to the results for daily returns. For each return interval, the minimum mean  $S_\beta$  occurs in the eight-year estimation period. However, as in the case of the daily return interval, most of the reduction has occurred by the third year. It should also be noted that for any given estimation period, the  $S_\beta$  increases monotonically with the return interval.

The test results with actual data corroborates the simulated results. First, given identical return periods, shorter return intervals are associated with smaller  $S_\beta$  or greater precision in estimating beta. Again, financial managers should use daily data to estimate beta. Second, increasing the estimation period from one year to three years

substantially decreases the  $S_\beta$  or increases in the precision of the estimated betas. The three-year estimation period captures 91 percent of the maximum reduction obtainable by increasing the estimation period from one year to eight years.

**TABLE 3**  
**Actual mean standard errors of estimated betas as a function**  
**of the return interval and estimation period over the 1982 to 1989.**

Estimation Period (Years)	Return Interval							
	Daily		Weekly		Two-Weekly		Monthly	
	Mean # of Returns	Mean $S_\beta$	Mean # of Returns	Mean $S_\beta$	Mean # of Returns	Mean $S_\beta$	Mean # of Returns	Mean $S_\beta$
1989 (1)	246.1	0.2826	48.8	0.5341	23.9	0.8135	12.0	0.6918
1988-1989 (2)	493.5	0.1723	99.6	0.3680	49.8	0.4613	24.0	0.4779
1987-1989 (3)	743.7	0.0823	150.5	0.1796	75.3	0.1996	36.0	0.2522
1986-1989 (4)	994.1	0.0774	201.5	0.1641	100.8	0.1877	48.0	0.2100
1985-1989 (5)	1242.8	0.0734	252.2	0.1529	126.2	0.1748	60.0	0.1946
1984-1989 (6)	1492.2	0.0692	302.9	0.1319	151.4	0.1685	72.0	0.1812
1983-1989 (7)	1743.4	0.0658	355.1	0.1255	177.1	0.1479	84.0	0.1762
1982-1989 (8)	1991.5	0.0621	405.2	0.1154	202.0	0.1348	96.0	0.1635

$S_e$  is the estimated standard deviation of the errors from equation (1).

$S_m$  is the standard deviation of the market return.

### Tests for Non-stationarity in Beta

Longer estimation periods appear to give a slightly more precise estimate of beta if the firm's fundamental structure does not change. However, many firms have seen a shift in their structure, which changes the firm's systematic risk. Thus, if beta is not stationary, longer estimation periods may lead to biased estimates of beta.

To test whether the firm's beta has shifted when one additional year of data is added to the estimation period, the following equation is estimated for each firm using 1988-1989 returns:

Equation 3

$$R_{it} = \alpha_i + \gamma_{1988i}D_{1988} + \beta_i R_{mt} + \Delta_{1988i}D_{1988}R_{mt} + \varepsilon_{it}$$

where  $R_{it}$  is the stock return on firm  $i$  in period  $t$ ;  $R_{mt}$  is the CRSP equally-weighted market return in period  $t$ ;  $\alpha_i$  is the intercept for firm  $i$  in 1989;  $\beta_i$  is the beta for firm  $i$  in 1989;  $\varepsilon_{it}$  is the error for firm  $i$  in period  $t$ ;  $D_{1988} = 1$  for observations in 1988, and 0 otherwise;  $\gamma_{1988i}$  is the intercept for firm  $i$  in 1988; and  $\Delta_{1988i}$  may be interpreted as the shift in beta by adding the 1988 daily returns to the estimation period. If  $\Delta_{1988i}$  is significantly different from zero, then the beta for 1988-1989 is significantly different from the beta of 1989.

For firm's with a stationary beta, we test whether the firm's beta shifts when a second year of daily returns is added to the estimation period. We estimate the following equation with 1987-1989 daily returns:

Equation 4

$$R_{it} = \alpha_i + \gamma_{1988i}D_{1988} + \gamma_{1987i}D_{1987} + \beta_i R_{mt} + \Delta_{1987i}D_{1987}R_{mt} + \varepsilon_{it}$$

Again, the intercept is allowed to vary in each period. Additionally  $\beta_i$  may be interpreted as the beta for 1988-1989 and  $\Delta_{1987i}$  may be interpreted as the shift in beta by adding the 1987 daily returns to the estimation period. If  $\Delta_{1987i}$  is significantly different from zero, then the beta for 1987-1989 is significantly different from the beta of 1988-1989.

This procedure continues, estimating equation (4) for the firms that had stationary betas in the previous iteration with intercept dummies for each period. Table 4 reports the results. The period over which beta is assumed constant and the test period appear in column one. The number of firms for which the estimation is performed appears in column two (this is the number of firms that had stationary betas in the previous iteration). The mean absolute shift,  $|\Delta|$ , and the percent of the sample with significant  $\Delta$  appear in columns three and four. The cumulative percent of the original sample of 1329 firms that have non-stationary beta over the estimation period appears in column five.

The tests results support that non-stationarity of beta is an important consideration in choosing the length of the estimation period. If one year of daily returns are added to the estimation period, then 36 percent of the firms have a significant shift in beta. If two years of daily returns are added to the estimation period, then 47 percent of the firms have a significant shift in beta. If seven years of daily returns are added to the estimation period, then 86 percent of the firms have a significant shift in beta.

## SUMMARY

Financial managers can estimate the cost of equity via the CAPM approach. If the financial manager estimates the firm's beta via regression analysis, then the financial manager must select both the return interval and the estimation period. Regarding return interval, this study finds that the financial manager should always select daily returns because daily returns result in the smallest standard error of beta or greatest precision of the beta estimate. However, regarding estimation period, the financial manager faces a dilemma. While a longer estimation period results in a tighter standard error for the estimate of beta, a longer estimation period also results in a higher likelihood that there will be a significant change in the beta. Thus, the beta estimated over longer estimation periods is more likely to be biased and of little use to the financial manager. The results show that an estimation period of three years captures most of the maximum reduction in the standard error of the estimated beta from a one-year estimation period to an eight-year estimation period. Additionally, less than fifty percent of the firms experience a significant shift in beta over a three-year period.

**TABLE 4**  
Test results for non-stationarity in beta for firms over the 1982 to 1989 period.

$$R_{it} = \alpha_i + \sum \gamma_{ik} D_{ik} + \beta_i R_{mt} + \Delta_i D_j R_{mt} + \varepsilon_{it}$$

Stable Period versus Test Period	Number of Firms	Mean $ \Delta $	Stocks with Significant $\Delta$	Cumulative Stocks with Significant $\Delta$
89 vs 88	1329	0.40	.18	.18
89-88 vs 87	1085	0.33	.36	.47
89-87 vs 86	698	0.33	.22	.59
89-86 vs 85	544	0.38	.14	.65
89-85 vs 84	468	0.36	.24	.73
89-84 vs 83	357	0.34	.27	.80
89-83 vs 82	261	0.31	.28	.86

## ENDNOTES

1. The tests of this theory comprise a vast literature with mixed conclusions. For example, Fama and French [4] conclude that stock returns are not related to systematic risk, but instead are related to the firm's size and its ratio of book equity to market equity. In contrast, Kothari, Shanken, and Sloan [6] conclude that there is a significant risk premium associated with systematic risk.
2. See Altman, Jacquillet, and Levasseur [2], Baesel [3], Gonedes [5], Roenfeldt, Griepentrof, and Pflaun [7], Smith [8], and Alexander and Chervany [1].
3. It is possible to create monthly returns from the daily returns. However, if a financial manager uses monthly returns to estimate beta, it is likely that he would use monthly returns from a database such as CRSP.
4. The mean betas are very close to one. The procedure was replicated with returns defined as  $\log_e(1+\text{return})$  and the results are virtually identical.
5. A frequency count reveals that for most securities, the minimum  $S_B$  occurred in estimation periods of eight years. However, for each return interval there were some firms whose minimum  $S_B$  occurred in estimation periods as short as three years. Additionally, the decrease in the mean  $S_B$  is tested between all adjacent estimation periods and is found to be a significant decrease at the one percent level.
6. There is a positive correlation between the estimated  $\beta$  and the standard error of the estimate; i.e., firms with large  $\beta$ 's typically have less precision in the estimate of  $\beta$ . This is not surprising, since firms with large systematic risk usually have relatively large non-systematic risk. However, there is a systematic relationship between this correlation and the combination of return interval and estimation period. In general, the correlation increases as return interval increases and as the estimation period increases.
7. The larger-than-average market volatility during 1987 serves to enhance the precision of the estimated beta. The tests were repeated for the period 1987-1980 and the conclusion are virtually identical: estimation periods of three years are sufficient.

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