

A MODEL FOR OPTIMAL UTILIZATION OF A FIRM'S LINE OF CREDIT

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INTRODUCTION

Constant pressure to increase return on assets has firms seeking ways to reduce their working capital costs. In the cash management area, firms are employing more sophisticated collection and disbursement systems. Cash management systems today efficiently speed up collections and, at the end of the day, sweep excess balances into money market accounts. Cash managers focus on finding the optimal cash–short-term investment mix.

We can view cash as a raw material. Accordingly, the cash manager and the production manager face similar challenges. The production manager is responsible for maintaining appropriate levels of raw materials, work-in-progress, and inventories. Similarly, the cash manager is responsible for maintaining optimal cash balances.

Maintaining appropriate cash balances or inventory levels involves managing flows. As long as the cash manager has sources of credit (access to cash), the firm can cover operating costs while maintaining minimal cash balances. Likewise, the production manager who is able to purchase materials on an as needed basis can minimize the firm's inventory levels.

Inefficient use of cash and materials ultimately reduces the firm's profitability. Inadequate levels of cash can preclude a firm from meeting its financial obligations as they become due, while material shortages can prevent meeting production schedules. Excess levels of cash and inventories tie up capital and reduce the firm's return on assets.

Short-term assets such as cash and inventories represent investments a firm must make in order to support its operations. As Beranek [2] notes, companies have short-term assets only because they face uncertainties related to their operations. For example, a firm could incur substantial costs if the labor force of a vendor supplying a critical part suddenly went on strike. An inventory of the critical part enables the firm to continue operating while it seeks an alternate supplier or waits out the strike. Similarly, a firm may hold a cash reserve to meet unanticipated demand for cash. Since cash is an unproductive asset, cash reserves are often held in the form of highly liquid short-term investments.

Baumol [3] recognized the similarities between cash and inventory management. He extended the economic order quantity (EOQ) model to examine its implications to cash management. The Baumol model assumes the cash manager invests excess funds in interest bearing securities and liquidates them to meet the firm's demand for cash. As investment returns increase, the opportunity cost of holding cash increases and the cash manager decreases cash balances. As transaction costs (cost of liquidating short-term investments) increase, the cash manager decreases the number of times he liquidates securities, leading to higher cash balances.

Managing the cash - short-term investments mix involves determining the optimal frequency for replenishing cash and the amount of securities to liquidate. Liquidating short-term investments is costly.¹ Costs include the brokerage fees, delivery costs (such as the cost of a bonded courier), telephone charges, the opportunity cost of redirected managerial effort, and possibly, a liquidity discount on the securities the firm sells. A liquidity discount is the difference between a dealers quote and the price the firm would receive if it could wait for a buyer willing to pay full market value. Along with Baumol, Miller and Orr [6], and Stone [11] developed models for managing cash and short-term investments. The major limitations of cash management models are in their underlying assumptions. The Baumol model requires that cash outflows are certain and constant. The Miller-Orr model assumes that a firm's net cash flows are normally distributed with a zero mean and a constant standard deviation. The Stone model is similar to the Miller-Orr Model except it assumes the firm has some knowledge of its demand for cash. Mullins and

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Homonoff [7] found that these models perform fairly well (with modifications) and can reduce the amount of time managers allocate to monitoring cash balances.

The uncertainty associated with a firm's cash flows limits the domain within which cash management models work. An alternative approach that does not assume a given cash flow pattern is to forecast cash flow patterns as accurately as possible, sweep surplus cash into a short-term account, and meet unexpected demand for cash by borrowing from a line of credit. The credit line commonly has two costs, interest on the funds the firm borrow and a commitment fee that varies with the maximum amount available.

A cash manager can extend the usefulness of a line of credit by utilizing it as a cash management tool rather than just a source of standby funds. In this paper, we develop a model that employs the firm's line of credit to reduce the total cost of its cash management. The model extends the Baumol model to include meeting cash disbursements by borrowing from a line of credit. The model helps us to explore the tradeoffs between transactions costs (converting securities to cash) versus lost investment income (from the short-term investments portfolio) and interest expense (from credit line borrowing).

The interest rate on the line of credit normally exceeds the rate of return on the short-term investments portfolio. Hence, borrowing from the line of credit will result in an interest expense that exceeds the lost income associated with liquidating securities from the short-term portfolio. However, the line of credit has no transactions costs. Borrowing from the line of credit reduces the number of times a firm liquidates securities to replenish cash, thereby reducing transactions costs and lost investment income.² The line of credit (*LOC*) model solves for the optimal amounts of securities to convert to cash and borrow from the line of credit each cash cycle. The model helps us understand the transaction costs - lost interest income - interest expense tradeoffs. We also examine how understanding the tradeoffs could help a cash manager avoid taking a sizable liquidity discount on a security by borrowing from a line of credit. In the next section, we discuss the Line Of Credit model.

THE LOC MODEL

A portfolio of highly liquid, short term securities and a line of credit provide cash reserves for a firm. The interest rate on the line of credit normally exceeds the return on the short-term investment portfolio. Thus, the firm would tend to replenish the cash account by liquidating short-term securities rather than borrowing from the more expensive line of credit. The line of credit serves as a backup source of funds for meeting unexpected cash disbursements.

The Baumol model provides guidance as to how often the firm's cash manager should replenish the cash account from the short-term investment portfolio. To set up the model, the firm estimates the fixed cost of liquidating securities, the return on its short term investment portfolio and its periodic demand for cash. The Baumol model indicates the cash replenishing amount that minimizes the total cash management cost (the sum of lost income on short-term investments and transaction costs).

While the line of credit is a more costly source of funds than the short term investments portfolio, it has no transactions cost. By extending the Baumol model to include borrowing from the line of credit, we can reduce total cash management costs. With the Baumol model, the firm replenishes cash from its short term investment portfolio periodically and disburses it steadily until the cash balance reaches zero. When the cash balance reaches zero, the firm replenishes it again and the next cash cycle begins. To extend the cash cycle to include borrowing from the line of credit, we partition the cash cycle into two parts. The first part begins with replenishing cash from the short-term securities portfolio. When the cash balance reaches zero, the firm begins to meet its demand for cash by borrowing from its line of credit. After meeting its cash disbursements from its line of credit for a predetermined period, the firm liquidates a sufficient amount of securities to repay the borrowing from the line of credit and replenish the cash account. We illustrate this cycle in Figure 1.

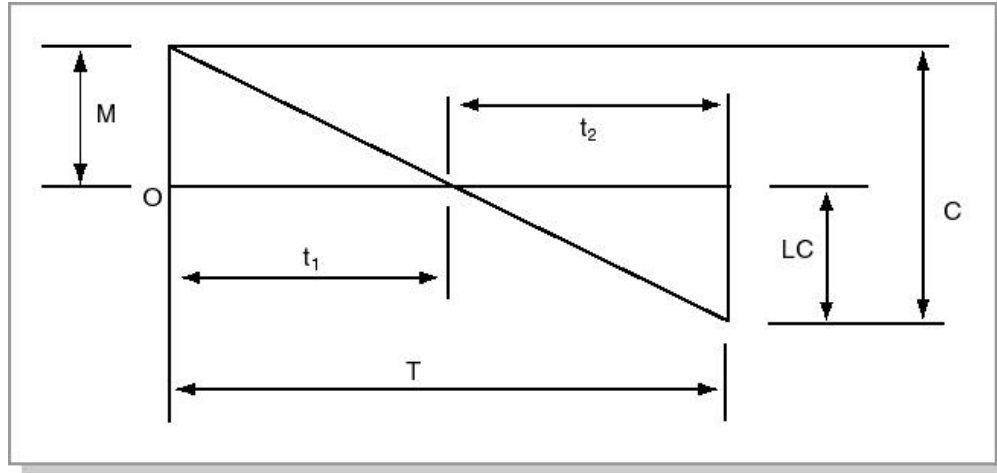
In order to find the optimal maximum cash balance and amount to borrow from the line of credit, we make some of the same simplifying assumptions that apply to the Baumol model:

- the firm can forecast future cash outflows with certainty
- the cost to convert short-term investments to cash does not vary with the dollar amount the firm converts
- a continuous cash disbursement rate.

The firm begins the cycle with a cash account balance of " M " dollars. It pays out cash steadily until its balance is zero (part t_1 of the cycle). At that point, the firm begins meeting its cash disbursements by borrowing from its line of credit (part t_2 of the cycle). After the firm has borrowed " LC " dollars from its line of credit (paid out a total of $M +$

$LC = C$ dollars), it converts C dollars of short-term investments to cash. With these funds, the firm replenishes the cash account balance to M dollars and repays the funds it borrowed from the credit line.³ With the cash balance again at M , the next cycle begins.

FIGURE 1
Cash Flow Cycle



- C dollar value of short-term investments the firm liquidates each cycle
M maximum cash balance
LC maximum dollar amount the firm borrows from its line of credit
 t_1 part of the cash cycle in which the firm meets its demand for cash from short-term investments it liquidates
 t_2 part of the cash cycle the firm meets its demand for cash by borrowing from its line of credit
T cash management cycle. This is the sum of t_1 and t_2 .

As an alternative to borrowing from its line of credit during part t_2 , the firm could attempt to obtain credit from its suppliers. Sastry [10] developed a similar model in which the firm accumulates obligations (by methods such as stretching payables) during part " t_2 " of Figure 1. Estimating the cost of accumulating obligations is difficult. Creditors may charge various penalties on overdue accounts. In order to accumulate obligations, the firm may be forced to forego discounts. The cost in terms of supplier relationships and the firm's credit ratings also would be difficult to measure. A survey by Hill, Sartoris, and Ferguson [4] found that firms value their credit ratings and their relationships with suppliers. Borrowing from its line of credit during period part " t_2 " (rather than accumulating obligations) preserves a firm's relationships with its suppliers. Also, by complying with the terms of the line of credit, the firm can preserve and even strengthen its credit rating.

The annual cost of the cash management cycle in Figure 1 includes investment income the firm foregoes by holding cash, interest the firm pays on its line of credit, and transaction costs associated with liquidating its short-term investments to cash. The total cost (TC) is:

$$TC = \text{Lost Investment Income} + \text{Interest On Line Of Credit} + \text{Transactions Cost}$$

Equation 1

$$TC = \frac{M^2 K_1}{2c} + \frac{(C - M)^2 K_2}{2c} + \frac{SD}{c}$$

each cycle.

- k_1 is annual rate of return on the short-term investments portfolio.
- k_2 is the interest rate on the line of credit.
- $C - M$ is the maximum amount the firm borrows from the line of credit.

The dollar value of short-term investments the firm liquidates at the beginning of each cycle that results in the lowest total cost is:

Equation 2

$$C = \sqrt{\frac{2SD}{k_1}} \sqrt{\frac{k_1 + k_2}{k_2}}$$

By liquidating C dollars each cycle, the maximum cash balance (M) is (a full derivation of TC , C , and M is in the Appendix)

Equation 3

$$M = \frac{k_2 C}{k_1 + k_2}$$

As an example,⁴ the Aubrey Company has a daily demand for cash of \$4,000, or \$1,460,000 annually. Short-term investments earn 5% annually. The interest rate on Aubrey's line of credit is 8%, while the cost to convert marketable securities to cash is \$50 per transaction. The optimal value of C for each cash management cycle is:

Equation 4

$$C = \sqrt{\frac{2(50)(1,460,000)}{.05}} \sqrt{\frac{.05 + .08}{.08}}$$

The maximum cash balance is:

Equation 5

$$M = \frac{(.08)(68,884)}{.05 + .08} = \$42,390$$

The maximum amount Aubrey borrows from the line of credit each cycle is:

Equation 6

$$LC = C - M = \$68,884 - \$42,390 = \$26,494$$

On an annual basis, Aubrey will convert short-term securities to cash:

Equation 7

$$\frac{D}{C} = \frac{\$1,460,000}{\$68,884} = 21.2 \times$$

Aubrey will convert \$68,884 of securities to cash every 17 days in a 365 day year. With each \$68,884, the Aubrey's cash manager will replenish the firm's cash balance to \$42,390 and repay the \$26,494 Aubrey borrowed from the line of credit. Aubrey's total annual cost of the 21.2 cash management cycles is:

Equation 8

$$\frac{M^2 k_1}{2C} + \frac{(LC)^2 k_2}{2C} + \frac{SD}{C} = \frac{(42390)^2 (.05)}{2(68884)} + \frac{(26494)^2 (.08)}{68884} + \frac{50(1460000)}{68884}$$

This amounts to \$2270.

We compare the total cost of the line of credit (*LOC*) model with that of the Baumol model. To facilitate the comparison, we compute Aubrey's optimal *C* for the Baumol model. If Aubrey follows the cash flow cycle indicated by the Baumol Model, the optimal dollar value of the securities it liquidates each cycle is:

Equation 9

$$C = \sqrt{\frac{2SD}{k_1}} = \sqrt{\frac{2(50)(1460000)}{.05}} = \$54,037$$

Aubrey's total cost is:

Equation 10

$$TC = \frac{Ck_1}{2} + \frac{SD}{C} = \frac{(54037) \cdot .05}{2} + \frac{(50)1460000}{54037} = \$2,702$$

Table 1 provides a summary of the cash management costs for the Baumol and *LOC* models. With the *LOC* model, Aubrey foregoes \$652 interest from short-term investments (by holding cash), pays \$408 in interest on its line of credit and \$1,060 in transactions costs. Under the assumptions of the Baumol model, Aubrey foregoes \$1,351 interest from short-term investments and pays an equal amount in transactions costs. Thus, extending the Baumol model to include borrowing from the line of credit reduces Aubrey's cash management costs by \$2,702 - \$2,207 = \$495.

TABLE 1
Baumol Model and Line of Credit (*LOC*) Model Costs

ASSUMPTIONS:			
Annual Demand for Cash (<i>D</i>): \$1,460,000			
Fixed Cost per Transaction (<i>S</i>): \$50			
Interest Rate on Temporary Investments (<i>k</i> ₁): 5%			
Interest Rate on Line of Credit (<i>k</i> ₂): 8%			
	Baumol Model	<i>LOC</i> Model	Difference
Transactions Costs	\$1,351	\$1,060	\$291
Lost Interest Income	1,351	652	699
Interest on <i>LOC</i>	0	408	-408
Total	2,702	2,120	582

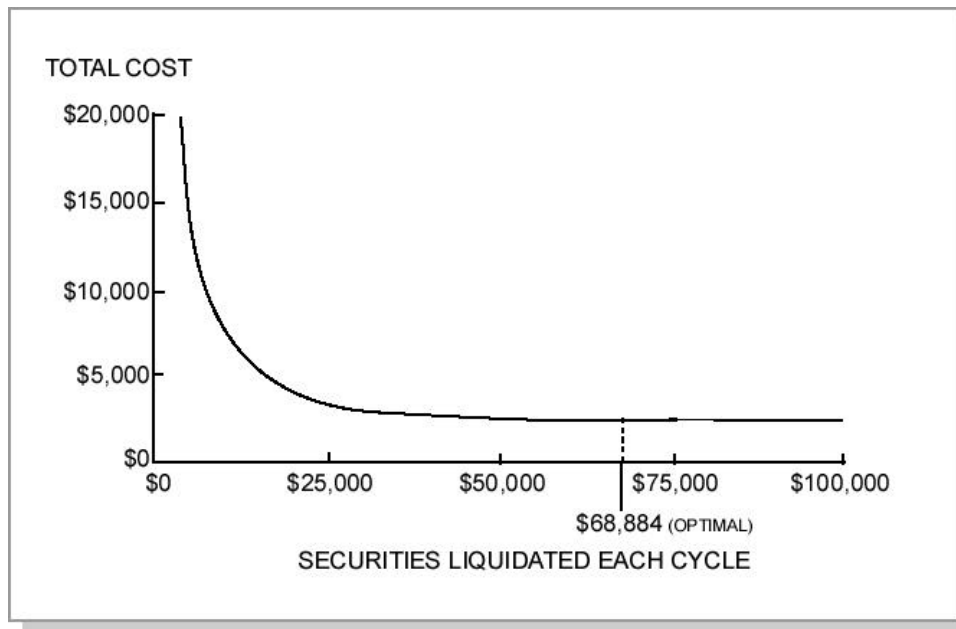
Having the same simplifying assumptions as the Baumol model restricts the domain of applications for the *LOC* model. In their discussion of the Baumol model, Maness and Zietlow (5) identify mortgage bankers, apartment rentals and health care providers, and governmental agencies receiving lump-sum reimbursements as organizations that fit the assumption of periodic cash receipts and continuous disbursements. These types of organizations would also be candidates for the *LOC* model. Organizations such as utilities and universities might also benefit from examining the *LOC* model.

A way of working around the periodic cash receipts and continuous disbursements is to shorten the modeling period from 1 year (as we did in the Aubrey Company example) to a quarter, a month, or even a week. As a firm updates its cash disbursements estimates, it can recalculate *C* and *M*.

Even with its simplifying assumptions, the Line of Credit model provides valuable insights as to the cash costs related to liquidating securities either too frequently (causing excessive transaction costs) or too infrequently

(causing excessive loss of investment income and excessive interest expense). Figure 2 shows the relationship between the total cost (TC) and the dollar value of the securities the firm converts to cash each cycle. For the Aubrey company, liquidating securities in amounts less than \$25,000 each cycle results in a steep increase in cash management costs (when compared to converting the optimal amount \$68,884). Converting \$100,000 of securities each cycle results in a relatively smaller cost increase.

FIGURE 2
Total Cash Management Cost Versus Securities Liquidated Each Cycle



Changes to LOC model variables affect the amount of short-term investments Aubrey liquidates each cash cycle. Sensitivity analysis identifies which variables have the largest impact. In Figure 3, we vary the \$50 fixed cost of converting securities to cash (F) from -15% to $+15\%$ (\$42.50 to \$57.50), the 5% return on the short-term investments portfolio (k_1) from -15% to $+15\%$ (4.25% to 5.75%), and the 8% interest rate on the line of credit (k_2) from -15% to $+15\%$ (6.8% to 9.2%) while holding all other variables constant. As F increases, Aubrey increases the amount of securities it liquidates (C) each cycle while decreasing the number of times it liquidates each year. Aubrey maintains a higher cash balance (M) and borrows more from its line of credit (LC). As F decreases, Aubrey liquidates more frequently, decreases its cash balance, and decreases the amount it borrows from its credit line.

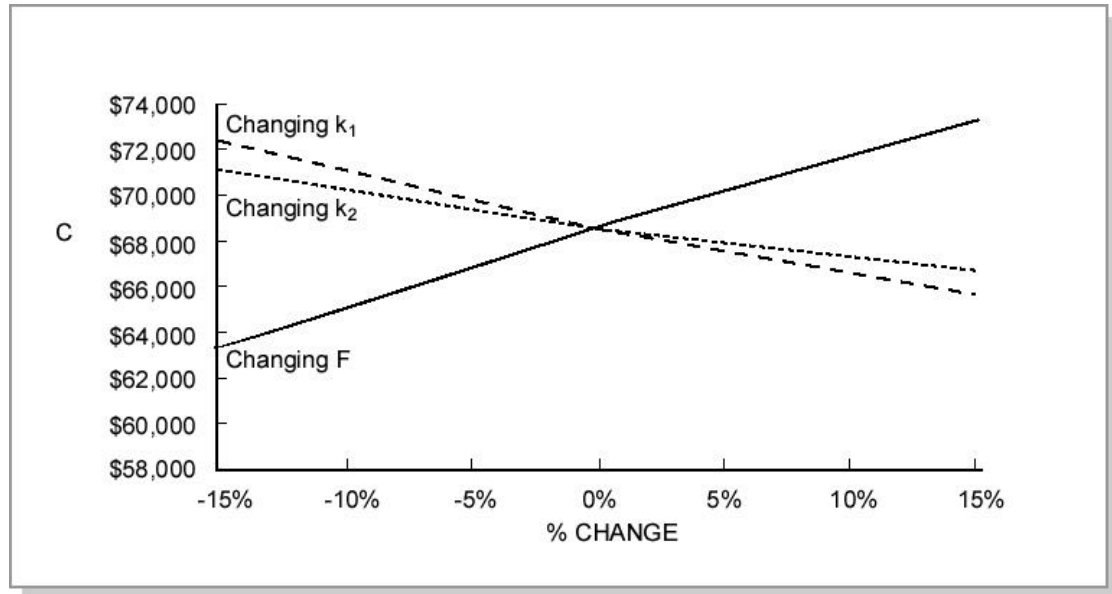
A zero per-conversion cost reduces the cash and securities mix problem to one of minimizing the cash balance. A firm utilizing a sweep account or a money market account (that does not have an explicit charge for each transfer to or from the cash account) would transfer funds on a daily basis to its disbursement account. The daily cash replenishing amount would be sufficient to meet each days cash disbursements plus, if required, a compensating balance amount.

As the return on the short-term investments portfolio increases, the opportunity cost of holding cash increases. Aubrey will convert securities to cash more frequently while reducing its maximum cash balance. As k_1 increases (holding k_2 at 8% and F at \$50), the relative cost of borrowing from the line of credit decreases. Thus, Aubrey will increase its line of credit borrowing.

The maximum amount Aubrey borrows from the line of credit decreases as k_2 increases (holding k_1 at 5% and F at \$50). As the cost of borrowing from the line of credit increases, Aubrey decreases the amount it borrows and decreases the amount of securities it liquidates each cycle (liquidating more frequently). As k_2 increases, the value of M approaches that of C . With an infinitely large k_2 , C equals M and also equals C from the Baumol model.

Of the three variables k , and F , the value of C is most sensitive to changes in F . In Figure 3, the change in C for each 5% change in F exceeds the change in C for a 5% change in either k_1 or k_2 . Thus, F is the most critical variable in the analysis of factors that cause changes in the cash balance.

FIGURE 3
Sensitivity of the Amount Securities the Firm Liquidates to Changes in F, k_1 and k_2



The LOC model shows the tradeoffs between transactions costs, lost interest income, and interest expenses. Understanding these tradeoffs can help reduce cash management costs outside the context of the model. As an example, Varun Inc. maintains a short-term investment portfolio. Varun's cash manager is considering selling commercial paper from its portfolio to meet a \$5 million financial obligation that is due today. The commercial paper matures in 70 days. The cash manager anticipates having sufficient cash flows from operations to meet the obligation in three days. Today, the secondary market for commercial paper is soft. Selling today, Varun Inc. would absorb a 40 basis point discount on the \$5 million of commercial paper. Including a \$25 brokerage fee, the cost of liquidating is:

Equation 11

$$\$5,000,000 \times (70/365) \times .004 + \$25 = \$3,861$$

Needing the cash for only 3 days, an alternative to liquidating the commercial paper is borrowing from the line of credit. If the line of credit has a 6.5% annual interest rate, the cost of borrowing for 3 days is:

Equation 12

$$\$5,000,000 \times (3/365) \times .065 = \$2,671$$

Resulting in a $\$3,861 - \$2,671 = \$1,190$ savings for Varun Inc.

The cash manager would evaluate the advantage (or disadvantage) of borrowing from the line of credit rather than liquidating short-term investments on a case-by-case basis. In Table 3, we calculate the cost of liquidating the \$5,000,000 of commercial paper with basis point losses of 5 to 50 points. For a 5 basis point discount, the loss Varun Inc. would take a by liquidating is:

Equation 13

$$\$5,000,000 \times (70/365) \times .0005 + \$25 = \$504$$

resulting in a $\$2,671 - \$479 = \$2,167$ savings compared to borrowing for three days from the line of credit.

TABLE 2
Liquidity Discounts Versus Cost of Borrowing from Line of Credit

Basis Points Loss	Transaction Cost Plus Liquidity Discount	Advantage (Disadvantage) to Using Line of Credit
5	\$504	(\$2,167)
10	\$984	(\$1,687)
15	\$1,463	(\$1,208)
20	\$1,943	(\$728)
25	\$2,422	(\$249)
27	\$2,671	0
30	\$2,902	\$230
35	\$3,381	\$710
40	\$3,861	\$1,189
45	\$4,340	\$1,669
50	\$4,820	\$2,148

CONCLUDING REMARKS

The line of credit (*LOC*) model extends the Baumol model to include borrowing from a line of credit. It is most applicable to a firm that experiences steady cash outflows and fixed transaction costs. With the *LOC* model, the firm's cash management strategy follows a two-part cycle. The cycle begins with the firm liquidating a specified amount of short-term investments. When the cash balance declines to zero, the firm meets its cash disbursements by borrowing a specified amount from its line of credit. After the firm's has borrowing reach the specified amount, it again converts short-term investments to cash. With these funds, the firm repays the amount it borrowed from its line of credit and replenishes its cash balance. When compared to the Baumol model, this strategy has a lower total cost.

A firm with quick access to money market loans can sell commercial paper to meet unexpected demand for cash. Commercial paper can also replace the line of credit in Figure 1. During period t_2 in Figure 1, the firm issues commercial paper (instead of borrowing from its line of credit) to meet its demand for cash. We would calculate the values of C and M in the *LOC* model substituting the cost of commercial paper for the interest rate on the line of credit (k_2).

For a firm with cash flow patterns that do not approximate those the *LOC* model assumes, the model helps understand the transactions cost – lost investment income – interest expense tradeoffs. In some situations, the firm should consider meeting part of its demand for cash by borrowing from the credit line rather than liquidating short-term investments, even though the interest rate on the line of credit normally exceeds the return on short-term portfolio. Understanding the tradeoffs will help to identify these situations.

ENDNOTES

1. Firms with accurate forecasts and access to money market loans are able to have zero balances. See Norgaard [8].
2. A firm's exposure to liquidity discounts is also reduced. See Amihud and Mendelson [1].
3. The short-term portfolio is replenished from operating cash flows.
4. We draw heavily on the example from a previous paper published in the *TMA Journal*.

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APPENDIX

To develop equations for the optimal levels of C and M , we will use the following definitions:

- k_1 is the investment rate of short-term investments.
- k_2 is the interest rate on the line of credit.
- D is the annual demand for cash (known with certainty).
- S is the fixed cost to liquidate short-term investments.

The opportunity cost of holding cash during period t_1 , is the average cash balance times the interest rate on short-term investments:

$$\frac{Mk_1t_1}{2}$$

The cost during period t_2 is the average amount the firm borrows from its line of credit times the interest rate on the line:

$$\frac{LCt_2k_2}{2}$$

The cash management cost during one cycle T is:

$$\frac{Mt_1k_1}{2} + \frac{LCt_2k_2}{2} + S$$

With an annual demand for cash of D , the firm converts short-term investments to cash D/C times per year. Thus the cash management cost per year is:

$$\frac{D}{C} \left[\frac{Mk_1t_1}{2} + \frac{LCk_2t_2}{2} + S \right]$$

From Figure 1, the following relationships hold (from similar triangle rules)

$$\frac{t_1}{T} = \frac{M}{C}$$

and,

$$\frac{t_2}{T} = \frac{LC}{C}$$

Since $T = C/D$ and $LC = C - M$, equation-1 can be rewritten:

$$\frac{M^2K_1}{2C} + \frac{(C - M)^2k_2}{2C} + \frac{SD}{C}$$

The annual cost of the cash management strategy in equation 2 includes interest the firm foregoes by holding cash, the interest the firm pays on its line of credit, and the transaction costs associated with converting its short-term investments to cash. To find the optimal levels of C and M , we differentiate equation 2 with respect to C and M :

$$\frac{\partial}{\partial C} = -\frac{M^2K_1}{2c^2} + \frac{k_2}{2} - \frac{M^2k_2}{2c^2} - \frac{SD}{c^2}$$

$$\frac{\partial}{\partial M} = \frac{Mk_1}{C}k_2 + \frac{Mk_2}{C}$$

Setting equation 4 equal to zero and solving for M :

$$M = \frac{k_2C}{k_1 + K_2}$$

To find the optimal dollar value of securities to convert to cash each cycle T , we set equation 3 equal to zero and substitute M from equation 4 into equation 3:

$$C = \frac{\sqrt{2SD(k_1 + k_2)}}{k_1k_2}$$

$$C = \sqrt{\frac{SD}{k_1}} \sqrt{\frac{k_1 + k_2}{k_2}}$$